Waveplate Retardance Metrology: The Basics and Beyond

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1. INTRODUCTION

Waveplates are important for the control of the state of polarization and for the measurement of polarization of light. They are modifiers of polarization state but not of the degree of polarization. They do this in a manner described by a parameter called retardance. Most commonly the needed retardances are either half wave or quarter wave. The accuracy to which these retardances are met by the waveplate determines the purity of the desired polarization state produced on output. Unfortunately, waveplate manufacturers and waveplate users often do not have an easy and accurate way to measure this important retardance parameter. We discuss here the basics and limits of retardance metrology as well our capabilities and equipment here at Meadowlark Optics for this retardance metrology.

2. ELLIPSOMETRIC VS SPECTROMETRIC RETARDANCE MEASUREMENT

Ellipsometric retardance measurements determine the retardance of a waveplate by single-wavelength intensity measurements with the test part between rotating polarizers.[1] Its advantage is the high accuracy of less 0.0005 waves of retardance at 630 nm, the simultaneous measurement of fast axis orientation (and of optical rotation, if required), and MLO's capability to provide spatial retardance mappings. For the ellipsometric measurement the order of a multi-order waveplate must be determined separately by a different method, typically by a spectrometer scan. The spectrometric retardance measurement method, on the other hand, determines the retardance together with its order, but less accurately, since it relies on known birefringence vs wavelength data of the sample material. For a compound waveplate the spectrometric method requires the layers to be of the same material and have aligned fast axes. The following table contrasts the two methods and their respective advantages:

	Ellipsometric method	Spectrometric method
Output raw data	Detected intensity	Spectrometer scan
Retardance accuracy (630 nm)	0.0002 waves	0.005 waves
Measured quantities	Retardance without order; fast axis; rotation	Retardance with order
Spatial mapping available	Yes	No
Station sold as MLO product	No	Yes
Multiple-material waveplates	Yes	No
Compound waveplates	Yes	Only for aligned axes
Time per measurement	1.5 - 3 min	1 min

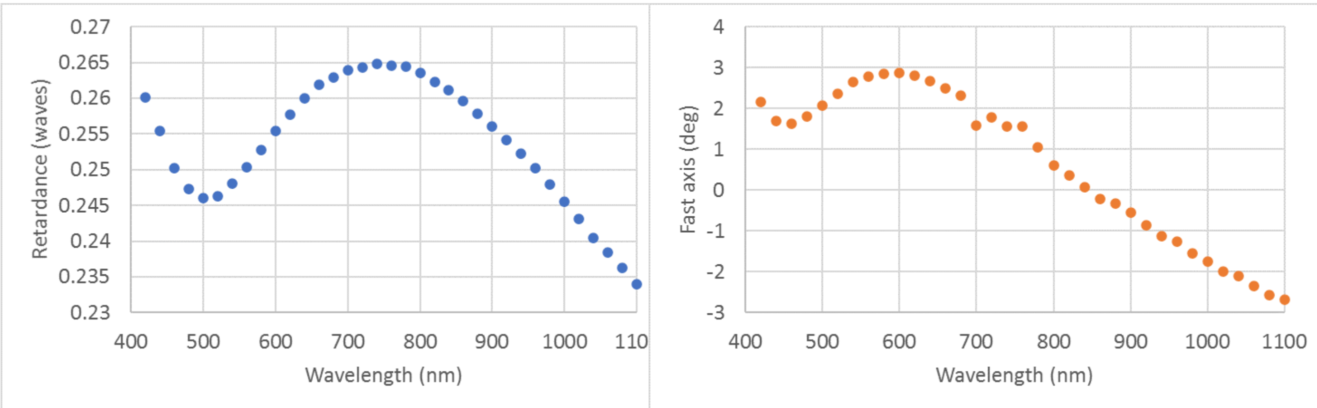
3. RETARDANCE SCANS

3.1 For pure retarders (retardance and fast axis vs wavelength)

The use of software-controlled monochromators allows MLO to consecutively measure retardance and fast axis orientation of a waveplate at different wavelengths for the purpose of

(a) evaluating the performance of different types of achromatic retarders,[2] see Fig. 1,

(b) measuring the birefringent dispersion of a single waveplate of a certain material to convert a retardance measured at one wavelength to the corresponding retardance at another wavelength. See Fig. 2.



(a) Retardance vs wavelength (b) Fast axis vs wavelength

Figure 1: Retardance scan for a 5-layer quarter-wave superachromat manufactured by MLO.

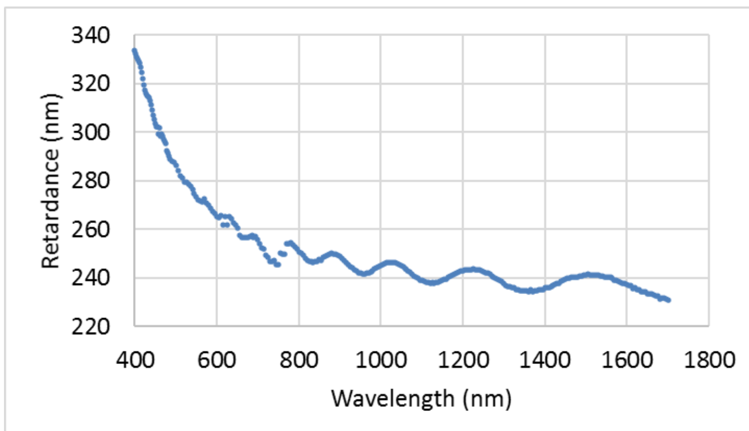


Figure 2: Retardance scan for a 2 microns thick reactive mesogen film, using separate retardance measurement every 5 nm (279 datapoints). Note the fringing caused by multiple reflections.

3.2 For elliptical retarders (retardance, fast axis and optical rotation vs wavelength)

In compound waveplates such as achromats or compound zero-order crystal waveplates, a small misalignment of plate fast axis directions can cause

(a) additional fast axis variations with wavelength,

(b) undesired optical rotation besides the intended retardation.

The retarder is, then, a so-called elliptical retarder, which - as proven by Hurwitz and Jones[3] - acts like the combination of a rotator and a retarder. With proprietary ellipsometric retardance measurement methods MLO measures retardance, fast axis direction and optical rotation for monochromatic light and converts these 3 measured quantities to linear and circular retardance components.[4] Elliptical retardance is desired for twisted-nematic (TN) liquid crystal cells as well as for modulator devices, designed to produce a large variation (as functions of wavelength or applied voltage) of output Stokes vectors, see Fig. 3.

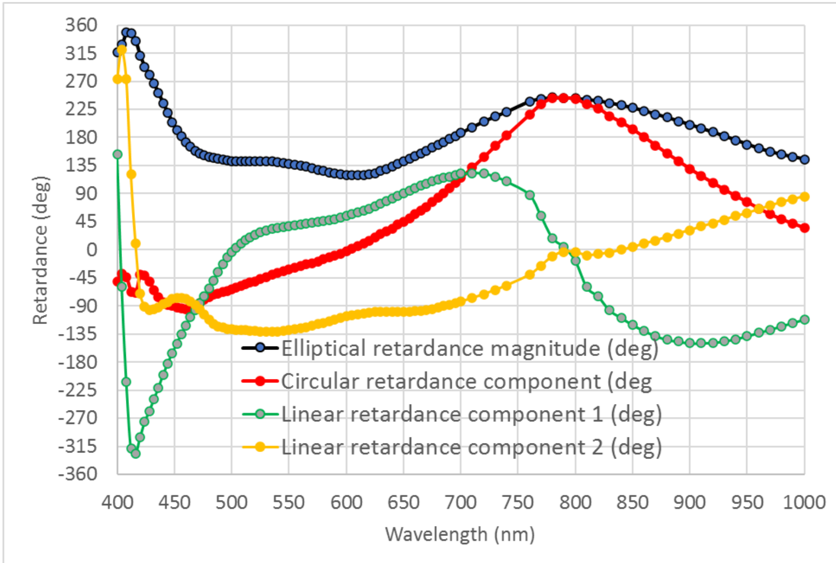


Figure 3: Elliptical retardance scan results for a polarization modulator built by MLO.

4. THERMAL DEPENDENCE OF RETARDANCE

Table 1 below list the thermal dependence of retardance for the three most commonly used crystals for waveplates, quartz, magnesium fluoride and sapphire, respectively. Our measurements are in good agreement with those of NIST.[5] The quantity γ is the fractional change in retardance with temperature per degree Celsius and retardance change is given by:

$$\frac{d\delta}{dT} = \gamma \cdot \delta \tag{1}$$

as defined by Etzel[5] where δ is the retardance and T is temperature. The retardance change depends on both the coefficient of thermal expansion as well as the thermal dependence of birefringence.

Table 1: Thermal retardance coefficients for crystal waveplates

(a) Quartz

WL(nm)	Quartz γ	Source
517	-1.03×10^{-4}	MLO
632.8	-1.213×10^{-4}	NIST
787.63	-1.196×10^{-4}	NIST
1064	-1.156×10^{-4}	MLO
1318.2	-1.25×10^{-4}	MLO
1538.89	-1.25×10^{-4}	MLO
1573	-1.142×10^{-4}	MLO

(b) Magnesium fluoride

WL(nm)	MgF_2 γ	Source
517	-4.80×10^{-5}	MLO
632.8	-5.55×10^{-5}	NIST
788.73	-5.03×10^{-5}	NIST
1064	-4.70×10^{-5}	MLO
1318.2	-5.95×10^{-5}	MLO
1538.89	-5.05×10^{-5}	MLO
1573	-4.72×10^{-5}	MLO

(c) Sapphire

WL(nm)	Sapphire γ	Source
517	-1.354×10^{-4}	MLO
1064	-1.202×10^{-4}	MLO
1573	-1.180×10^{-4}	MLO

We measure γ values for polymers to be roughly an order of magnitude higher than for these crystals except for PMMA which is roughly a factor of 100 times greater than for these crystals. The value is positive in sign for polycarbonate films, and polyvinyl alcohol films but is negative for polystyrene and PMMA films. For nematic liquid crystals used in variable retarders we measure γs in the range of $1-2\times10^{-3}$.

5. LIQUID CRYSTAL VARIABLE RETARDERS

Liquid crystal variable retarders have a retardance that is easily controlled by an AC square wave voltage usually of a frequency of one to two kHz. The retardance is a nonlinear function of applied voltage as shown in Figure 4. The retardance is slightly elliptical at voltages below four volts as indicated by the deviation from the liquid crystal alignment direction (slow axis direction) at zero degrees. This is also shown in Figure 4.

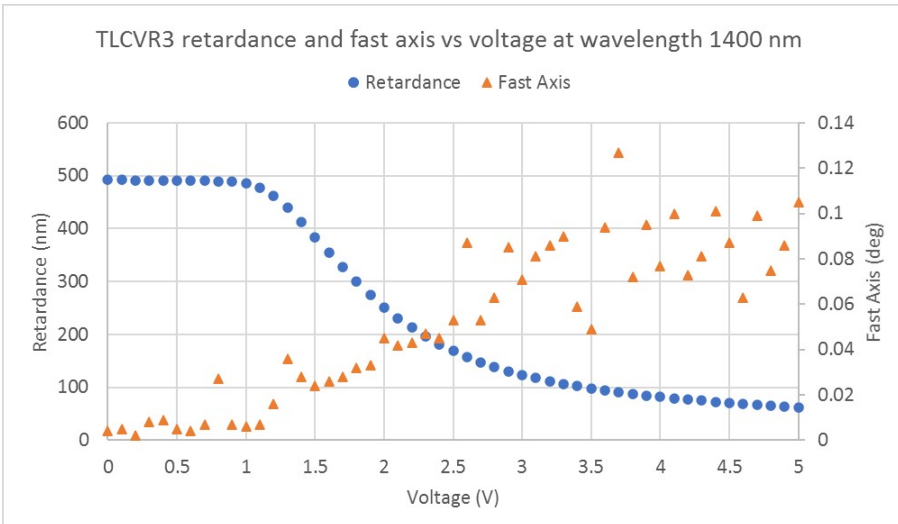


Figure 4: Retardance and fast axis rotation in a liquid crystal variable retarder.

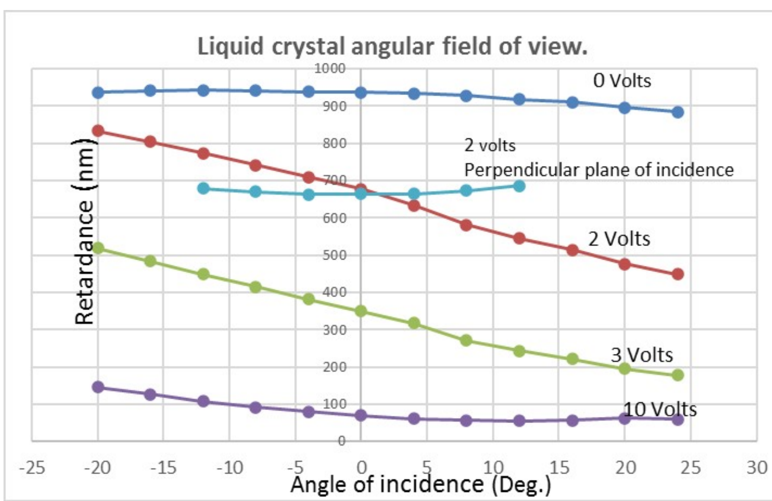


Figure 5: Measured retardance depends strongly on angle of incidence when the plane of incidence contains the liquid crystal alignment direction and the molecular tilt is Intermediate between 0 and 90 (2 and 3 volts). There is symmetry when the plane of incidence is perpendicular to the alignment direction (short curve).

Liquid crystal variable retarders have an asymmetric angular dependence of retardance. This is because the retardance variation with voltage results from electrical control of the effective birefringence of the liquid crystal layer. The applied voltage induces a dipole in the rod -shaped liquid crystal molecules and the electric field then applies a torque to the molecules tipping them out of the plane of the thin liquid crystal layer. The effective birefringence then depends on the angle between the incident ray and the axis of the liquid crystal molecule. Figure 5 shows quantitively the measurement of this effect for one liquid crystal cell. This effect is most pronounced when plane of incidence is parallel to the alignment direction of the liquid crystal molecules. The asymmetry is not present for rays in a plane of incidence perpendicular to this alignment direction as is also shown in the measurements in Figure 5.

6. SPATIAL MAPPINGS

6.1 Transmitted wavefront error

Figure 6 shows the transmitted wavefront error for a polycarbonate film that is 5.4 cm in diameter. The error is 0.41 waves peak to valley at a wavelength of 632.8 nm. This corresponds to a thickness variation of 0.26 microns which is 0.35% of the 75 micron film thickness. Usually we reduce the wavefront error by laminating the film between antireflection coated optically flat windows to achieve a reduction to a wavefront error of better than 0.125 waves. This has the additional advantages of:

1. stabilizing the retardance which will otherwise relax slightly over time and
2. minimizing retardance and transmission fringes, as also discussed below in section 7.

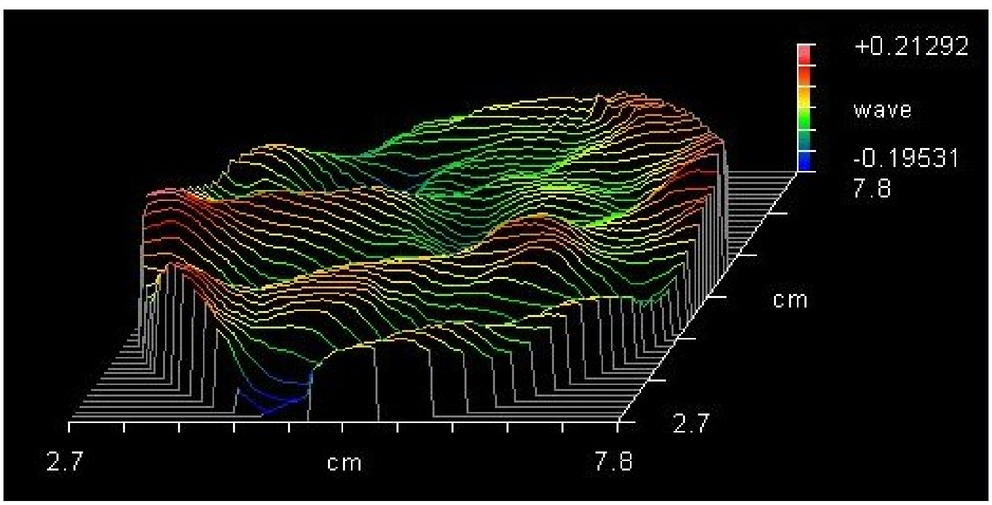


Figure 6: Transmitted wavefront error plot for a 5.4 cm polycarbonate film.

6.2 Retardance maps

There are spatial variations over clear aperture in all types of retarders. In polymers it is because of the thickness variations implied in the transmitted wavefront error map of Figure 6 as well as birefringence variations. Figure 8(a) shows the measured variation of retardance of a polycarbonate retarder over a clear aperture of 11 cm.[6] There is also a small variation in this retarder of fast axis variation of 0.5°, peak to peak. Figures 7 and 8(b) represent retardance maps of a compound magnesium fluoride and a compound quartz waveplate, respectively.

We find that the birefringence of quartz is not uniform within a single grown crystal. The single-plate quartz retarder[6] (4.233 mm thick) of Figure 8(b) was deterministically fluid jet polished to achieve a transmitted wavefront error at a wavelength of 632.8 nm of less than 0.01 waves, peak to peak or about 12 nm. This corresponds to a physical thickness variation of 7.7 nm assuming all the wavefront error is produced by crystal thickness variations. This thickness variation implies a retardance variation of 0.07 nm or 0.04°, more than 25 times less than the measured retardance variation. We are left to conclude that either the index of refraction of the crystal is non-uniform or more likely that the birefringence is not uniform. This crystal non-uniformity may be the cause of the variation of published measurements of quartz birefringence. Meadowlark Optics can produce retardance maps on a square grid up to 15.5 cm x 11 cm large with meshsize down to 3 mm.

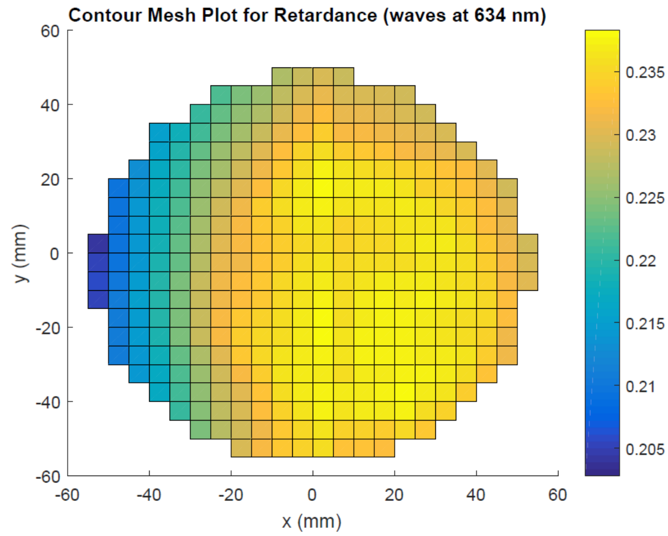
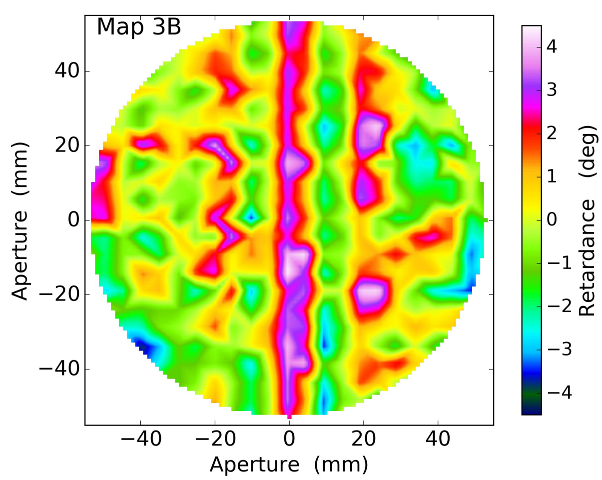
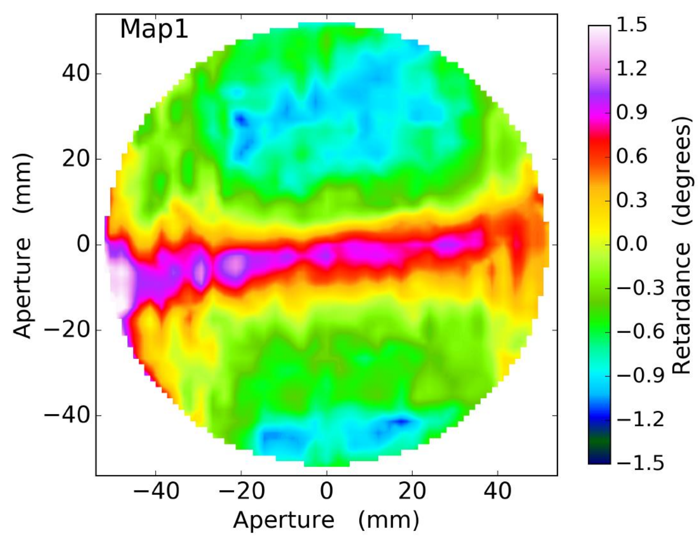


Figure 7: Retardance map of a 12 cm diameter magnesium fluoride compound waveplate.



(a) 11 cm diameter polymer retarder with an average retardance of 157.7° at 633 nm wavelength.



(b) 4.233 mm thick quartz waveplate measured at a wavelength of 631 nm.

Figure 8: Retardance maps, measured by MLO, are quoted from Figures 32(a) and 37(a) of SPIE JATIS, 6(3), 038001 (2020). https://doi.org/10.1117/1.JATIS.6.3.038001.[6] Notice: Re-printed under Creative Commons License CC BY 4.0, see https://creativecommons.org/licenses/by/4.0/ .

7. ETALON (FRINGING) EFFECTS IN WAVEPLATES

As examined by Holmes,[7] waveplates that are thin compared to the wavelength of monochromatic light propagating through them give rise to coherent multiple reflections, which lead to "fringes" in retardance as well as transmitted intensity. Fringe amplitude is reduced by anti-reflection coating of the waveplate. Figure 2 from above shows retardance fringes for a reactive mesogen thin film, and Figure 9 transmission fringes for unpolarized(!) light where the oscillations for the two refractive indices are beating against each other.[8]

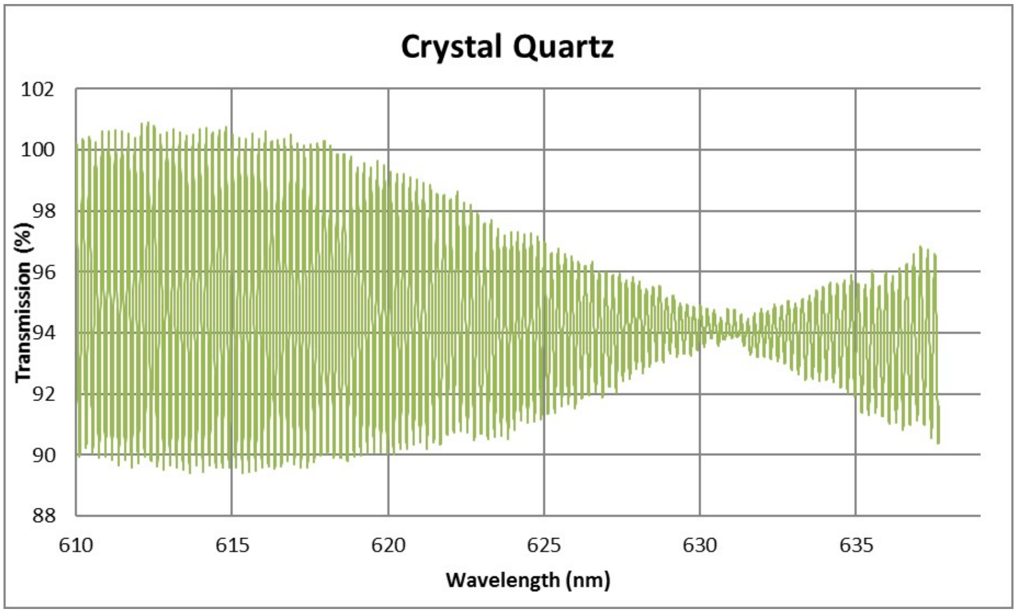


Figure 9: Measured wavelength dependence of transmission fringes in a crystal quartz retarder in unpolarized light. Attribution: Figure 8(b) from SPIE JATIS, 3(4), 048001 (2017). https://doi.org/10.1117/1.JATIS.3.4.048001.[8] Notice: Re-printed under Creative Commons License CC BY 3.0, see https://creativecommons.org/licenses/by/3.0/ .

8. CLOCKING OF COMPOUND ZERO ORDER CRYSTAL WAVEPLATES

A small clocking error in setting the rotational orientation between the two plates results in elliptical retardance when the desired retardance is pure linear. A wavelength variation of fast axis direction is an indicator of the presence of elliptical retardance. The measured amplitude of this periodic variation is shown in Figure 10 for an optically contacted quartz retarder made at Meadowlark Optics.[6] This amplitude is a measure of the amplitude of the clocking error. The clocking error in this case is 0.3°. A crystal waveplate from another vendor shows a larger measured clocking error of 0.8°. We have only done measurements of this error on these two waveplates but it is reasonable to expect similar errors on most commercially available compound zero order crystal waveplates. We have now developed techniques to reduce this clocking error to less than 0.1°.

9. SUMMARY

Retardance is the most important parameter in describing the polarization performance of waveplates and we have described two measurement methods and their accuracy limitations as well as our measurement capabilities here at Meadowlark Optics. Second order effects on waveplate performance include retardance variations with angle of incidence, spatial and thermal variations of retardance, clocking or rotational errors in compound waveplates and fast axis variations with wavelength in achromatic and liquid crystalwaveplates. Common waveplate materials include birefringent crystals, polymers and liquid crystals.

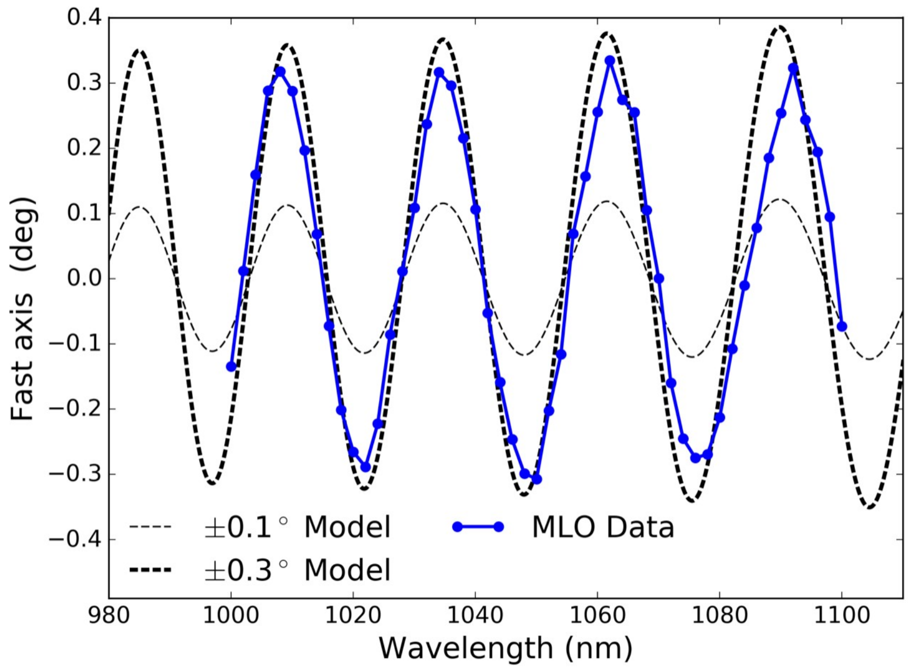


Figure 10: The measured wavelength variation of fast axis direction in a compound zero order retarder is shown in the solid blue curve. The model of the effect shows that the clocking error for this retarder is 0.3°. Attribution: Figure 19(a) from SPIE JATIS, 6(3), 038001 (2020). https://doi.org/10.1117/1.JATIS.6.3.038001.[6] Notice: Reprinted under Creative Commons License CC BY 4.0, see https://creativecommons.org/licenses/by/4.0/ .

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