Sources of Error in Retarders and Waveplates

Error sources that afflict true zero order, compound zero order and multi-order linear retarders include variations in temperature, angle of incidence, wavelength, and the presence of multiple reflections. This application note contains technical information designed to aid the user in producing the highest quality results and in making the best purchasing choice among retarder types for the intended application.

About linear retarders

A retarder, also known as a waveplate, is an optical device that modifies the polarization state of light by resolving it into two orthogonal linear polarization components and producing a phase shift between them. The phase shift results from the two polarization components having different velocities within the retarder material. Generally, waveplates are made from an uniaxial material which is one characterized by a single optic axis and two indices of refraction. The index of refraction along the optic axis is termed the extraordinary index, n_e, while that in the plane perpendicular to the optic axis is called the ordinary index, n_o.

The velocity of a component polarized along the optic axis, v_e, and one polarized perpendicular to it, v_o, is:

$$v_e = c/n_e \text{ and } v_o = c/n_o, \qquad (1)$$

where c is the speed of light in vacuum. By definition, a positive uniaxial material has a birefringence ($ß = n_e - n_o$) greater than zero. The slow axis is, therefore, along the optic axis for a positive uniaxial material while the fast axis is along it for a negative uniaxial material.

A retarder is typically made by placing the optic axis in the plane of the optic. The resulting retardance, in waves, is given by:

$$\delta = (n_e - n_o)t/\lambda = ßt/\lambda\ , \qquad (2)$$

where t is the optic's thickness and λ is the wavelength of light. Due to the wave nature of light, any integer number of waves of retardance leaves the polarization state unchanged. Thus, it is only the fractional part of the above which is responsible for polarization changes. This leads to three broad categories of retarders: multiple-order, true zero order and compound zero order.

Multiple order retarders have total retardation greater than 1, while that of a true zero order retarder is less than 1. Compound zero order retarders are composed of two multi-order components which differ by the desired retardation fraction and arranged with the fast axis of one parallel to the slow axis of the other so that they subtract from one another. The result is zero order.

Typical retarder materials are crystals (such as calcite, magnesium fluoride, and most commonly, quartz) and oriented polymers. It is nearly impossible to polish a true zero order retarder from traditional crystalline materials since the resulting parts are extremely thin. For example, a quarter wave retarder out of quartz (birefringence = .0092 in the visible) would be only 15 μm thick for light of wavelength 550nm. Thus, true zero order parts are usually made from polymers, while crystalline materials are used for multi-order and compound zero order pieces.

True zero order retarders are generally preferred for the most demanding applications requiring retardance stability since they are generally the least sensitive to variations in wavelength, angle of incidence and temperature. It is a common misconception that compound zero order parts share this stability, but compound zero order parts have the same sensitivity to angle of incidence as multiple order retarders of the same thickness. We describe the major errors and the sensitivity of each retarder type below.

Wavelength

Equation 2 above makes it obvious that retardation is a strong function of wavelength. In truth, the birefringence, ß, is also wavelength dependent, but contributes significantly less to the variation with wavelength than the explicit $1/\lambda$ dependence. The change of retardation as a function of wavelength can be described in the following equation:

$$\frac{d\delta}{d\lambda} = \delta\left(\frac{1}{\beta}\frac{d\beta}{d\lambda} - \frac{1}{\lambda}\right) \qquad (3)$$

As you can see, the change in retardation varies proportionally to the total retardation of the retarder. Therefore, retarders made of different materials have essentially the same wavelength dependence per wave of retardation. In other words a relatively thin ~0.7mm piece of quartz with a retardation of 11.25 waves is almost 50 times more sensitive than a true zero order quarter-wave polymer retarder. The retardation of a compound zero order retarder is given by:

$$\frac{d\delta}{d\lambda} = \delta_1\left(\frac{1}{\beta_1}\frac{d\beta_1}{d\lambda} - \frac{1}{\lambda}\right) - \delta_2\left(\frac{1}{\beta_2}\frac{d\beta_2}{d\lambda} - \frac{1}{\lambda}\right) = (\delta_1 - \delta_2)\left(\frac{1}{\beta}\frac{d\beta}{d\lambda} - \frac{1}{\lambda}\right)$$

In the final term above we assume that both elements of the compound retarder are made of the same material, in which case the variation is that of a true zero order retarder. Figures 1 compares the retardation change due to wavelength of true or compound zero order half wave retarders to a multi-order (~0.7mm thick) quartz retarder.

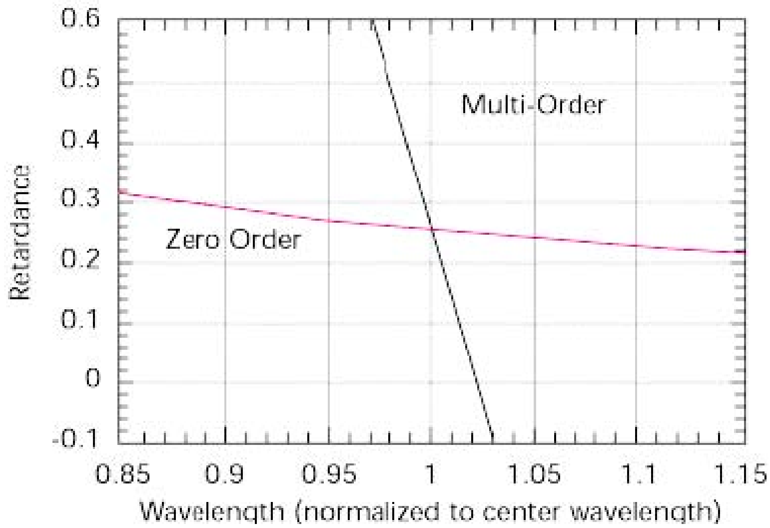

Figure 1: Comparison of variation with wavelength
of zero order and multi-order retarders.

Angle of Incidence

When the direction of propagation of the light is not precisely perpendicular to the optic axis, the second equation must be modified. The ordinary component will still lie in a plane perpendicular to the optic axis and have an index n_o. The extraordinary component, however, no longer lies along the optic axis, but is in an orthogonal plane which contains the optic axis. In this case, the correct index of refraction is given by:

$$n_{eff} = \frac{n_e n_o}{\sqrt{n_e^2 \cos^2(\theta) + n_o^2 \sin^2(\theta)}} = n_e\left(1 - \frac{\beta}{n_o}\cos^2(\theta)\right)$$

Here, θ is the angle between the optic axis and the direction of propagation in the material. The simplification, given in the final expression, results by assuming, as is almost always the case, that n_e-n_o is small compared to the individual indices. In addition if we limit ourselves to small angles we can ignore the difference in the direction of propagation between the two components due to refractive effects. In addition the effective thickness of the piece with ø, the angle made with the surface normal, increases as $1/\cos(ø)$ and

$$\delta = \frac{\beta t \sin^2(\theta)}{\lambda \cos(\varnothing)}$$

Two important special cases should be noted: 1) rotation about the optic axis (presumed to be in the plane of the optic) and 2) perpendicular to it. In the former case the effective extraordinary index remains unchanged, with $\theta = \pi/2$, and it is only the thickness which is affected:

$$\delta = \frac{\beta t}{\lambda} \cdot \frac{1}{\cos(\varnothing)} \qquad \text{rotation || to axis}$$

In the latter case both effects occur, $ø = \pi/2 \pm \theta$ with

$$\delta = \frac{\beta t}{\lambda} * \cos(\varnothing)$$

rotation $\perp$ to optic axis

Retardation magnitude increases with rotations parallel to the optic axis and decreases with rotations perpendicular to it. Unfortunately, unlike wavelength, (shown above), and thermal effects, (below), forming a compound zero order waveplate only exacerbates this problem. If we take a compound waveplate made of positive uniaxial materials so that the two optic axes are perpendicular to one another, then rotate about one optic axis, we necessarily rotate perpendicular to the other. The total change in retardation is:

$$\delta = \frac{\beta t_1}{\lambda} * \cos(\varnothing) - \frac{\beta t_2}{\lambda} * \frac{1}{\cos(\varnothing)}$$

Or, expanding the cosine in terms of ø for small angles:

$$\delta = \frac{\beta}{\lambda} t_1(1 - \frac{\varnothing^2}{2}) - t_2(1 + \frac{\varnothing^2}{2}) = \frac{\beta(t_1 - t_2)}{\lambda} - \frac{\beta(t_1 + t_2)}{\lambda} \frac{\varnothing^2}{2}$$

The second term in the last expression is simply the error in retardation of a single retarder of the same thickness as compared to the combined thickness of the compound zero order.

Figure 2 compares the field of view of a true zero order, multi-order retarder (11 orders) and a compound zero order retarder. It is possible to form a compound retarder out of one positive and one negative uniaxial material so that the optic axes are aligned and the retarder can exhibit excellent field of view. However, if other properties of the materials - such as thermal coefficients - are not matched, the gain in field of view can be negatively offset by other sources of error.

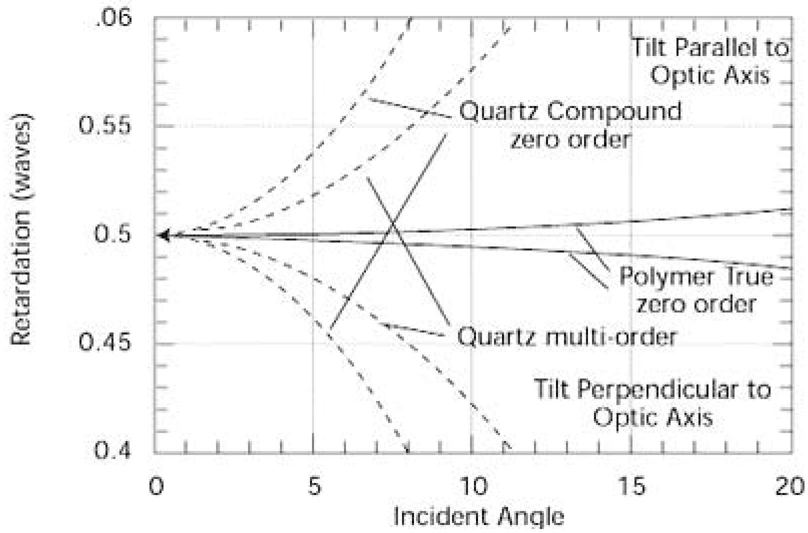

Figure 2: Comparison of affect of incidence angle on retardation for true zero-order, compound zero-order, and multi-order retarders.

Thermal

Retardation will change with temperature caused by a change in thickness of the retarder and/or a change in the birefringence of the material. The percentage change in retardation per °C ($r=1/\delta \; d\delta/dT$) for quartz is approximately 0.01% while for polymers it is typically 0.02% to 0.03%. Thus a true zero order quarter-wave polymer retarder will be roughly an order of magnitude more thermally stable than a good multi-order quartz retarder. A compound zero order retarder, made of two plates of the same material, will show the same thermal shift as a true zero order retarder of that material.

Coherence

If the coherence length of the light source is comparable to the thickness of the retarder, multiple reflections between the two surfaces can have a large - and negative - effect on retardation and transmission of the two polarization components. This is why it is critically important for waveplates used with laser light to have all surfaces coated with an AR coating.

Figure 3 shows an example of this etaloning effect for a retarder with bare surfaces. At the half wave point the oscillations approach zero, but at quarter wave they grow to over a hundredth of a wave in size and similar oscillations affect the relative amplitudes of the two polarization components. This can compound other error sources. (For example, a small temperature shift which would ordinarily change the retardation by less than a thousandth of a wave can be greatly amplified by the etaloning affect shown in Figure 3.)

The best way to remove this problem is to apply a high quality AR coat to all surfaces since the amplitude of the oscillations is proportional to the reflectivity of the surfaces. The period of the oscillations will depend on the precise retarder used.

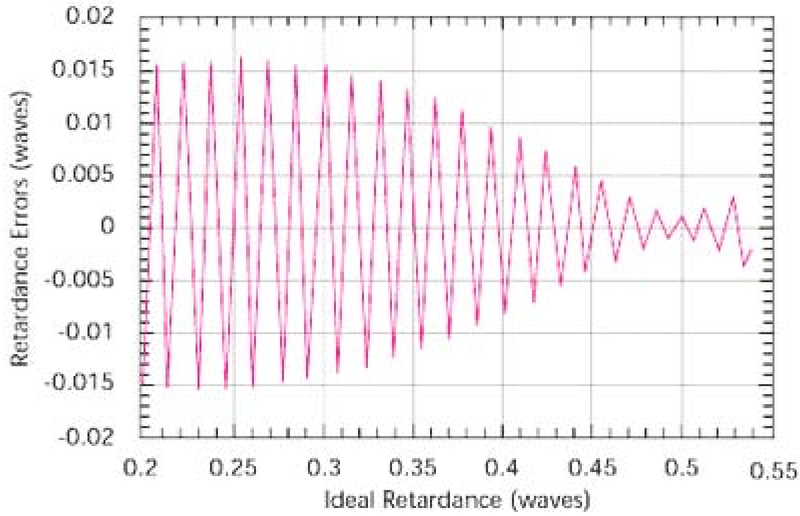

Figure 3: Retardance errors due to etaloning of coherent light in a retarder without AR coatings.

Summary

In summary, when choosing the best retarder for your particular application, a number of factors must be considered. These include not only the precision and cost of the retarder, but its susceptibility to the error sources it is likely to encounter in your application. The table below compares the key performance features of the retarders covered in this note.

	True Zero Order	Compound Zero Order	Multi-Order	Bi-Crystalline Achromats	Pancharatnam Achromats	Liquid Crystal Variable
Wavelength Stability	Good	Good	Poor	Excellent	Excellent	Good
Angle of Incidence Sensitivity	Excellent	Poor	Poor	Poor	Good	Good-Poor (dependent on voltage)
Thermal Stability	Good	Good	Poor	Fair	Excellent	Good
Power handling	Fair	Good	Excellent	Good	Fair	Good
Environmental durability	Fair	Good	Excellent	Good	Fair	Fair
Variable?	No	No	No	No	No	Yes
Cost	$$$	$$	$	$$$$	$$$$	$$$$

Pancharatnam achromatic retarders

We make this unique type of retarder using three to six layers of zero order polymer aligned with their optic axis at oblique angles. Developed by S. Pancharatnam roughly fifty years ago, these retarders have an extremely wide spectral range over which they remain essentially quarter- or half-wave. Such retarders are obviously insensitive to wavelength variation, even better than zero order waveplates, and are equally impervious to temperature variations. Being formed of a few sheets of zero order polymer means their field of view is only a few times worse than a true zero order polymer. However, due to the oblique alignment of the individual components, the exact field of view behavior is very complicated.

Liquid Crystal Variable Retarders

Meadowlark Optic's line of Liquid Crystal Variable Retarders (LCVRs) are formed by aligning a nematic liquid crystal between precision glass plates. An electric field applied to the optic adjusts the retardation by tilting the optic axis out of the plane of the optic. Our standard LCVRs have a maximum retardation less than one order. The variation with wavelength and temperature is thus equivalent to a true zero order retarder. The thermal variation caused by the liquid crystal material is negative, approximately -0.4%/°C. For rotations about the axis containing optic axes, its field of view is similar to that of a true zero order retarder. However, because of the tilted optic axis (when voltage is applied), rotations perpendicular to the optic axis produce large and voltage-dependent retardation changes. Changes as large as .01waves/degree are not uncommon.

Angle Tuning

A waveplate's retardation is a function of the angle of incidence. While this is normally an undesirable effect which limits the optic's numerical aperture and forces strict alignment, it can also be useful at times. If one needs a retarder with a slightly different retardation value, or must have a retardation value even more precise than Meadowlark's tight tolerances, angle tuning can be the solution. By rotating along the optic axis or perpendicular to it, the retardation can be finely controlled. Remember that in most optic setups the retarder axis will be aligned at 45° to the to the incoming polarization state of the light, generally horizontal or vertical. Thus, the tuning rotation will be made about a 45° axis.