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## **Liquid Crystal Optical Phase Modulators for Beam Steering**

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### **ABSTRACT**

Beamsteering using liquid crystals can be achieved with refractive or diffractive implementations. The common thread in these various structures is that the liquid crystal is employed as an optical phase modulator. Either nematic or smectic liquid crystal phases can be used to shift the phase of light and steer an optical beam. Various liquid crystal optical phase modulating schemes will be described. Examples include polarization independent and quasi-achromatic modulators. Model predictions and experimental results demonstrating the optical phase modulation and beamsteering made possible using different liquid crystal based designs will be presented.

### **INTRODUCTION**

Liquid crystals are frequently used as optical modulators. These materials offer several advantages including large modulation depth, no moving parts, low power dissipation, potential for large aperture operation, and low cost. Beamsteerers that employ liquid crystal modulators can be categorized according to the physical mechanism used to redirect light: refraction and diffraction. McManamon et. al. [1] provide an excellent review of liquid crystal beamsteering technology.

An example of refractive beam deflectors are liquid crystal wedges. In general, refractive beamsteerers offer high efficiency but small angular deflection. The deflection angle for a wedge is proportional to the optical path difference induced by electrically changing the effective refractive index of the liquid crystal. Consequently this approach yields deflection angles on the order of milli-radians. Cascading multiple elements in series can improve the deflection angle without drastically reducing the efficiency. Optical phased arrays can be used in refractive mode if no resets are used and the phase ramps continuously across the aperture. Since there are no phase resets, grating dispersion is not present and broadband radiation can be steered.

Diffractive beamsteerers can be implemented with an optical phased array analogous to some radar systems. Alternatively, the diffractive optical phased array can be thought of as a quantized multiple level phase grating. The more phase levels used in the array, the higher the diffraction efficiency. For example, a binary phase grating ideally provides a diffraction efficiency of 40.5% in each of the two first order diffracted beams. For a quantized phase grating using three phase levels the ideal first order diffraction efficiency is 68.4%, while for 4 phase levels, it increases to 81%. For more than four levels the improvement in diffraction efficiency with increasing number of phase levels slows. At 5 levels the percentage of light

diffracted into the first order is ideally 87.5% and for 8 phase levels the ideal first order diffraction efficiency is 94.9%.

Due to the effects of fringing fields between electrode lines, the actual phase profile is not a series of quantized steps but is smoothed such that the device more closely resembles a blazed grating. In addition to the phase profile, device efficiency also depends on the effective fill factor. The effective fill factor is governed by the size of the flyback region (where the phase reset defining a grating period occurs) relative to the size of the grating period [1]. Alternatively, one can attribute lower efficiencies at larger diffraction angles to a finite pixel (element) function. That is, the far field diffraction pattern for the grating written to the device is convolved with a pixel function which envelopes the far field such that efficiency decreases for larger angles.

The deflection angle for a diffractive beamsteerer,  $\theta_m$ , is given by

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) . \quad 1.$$

Here  $m$  is the diffracted order (usually only the first order is considered because it will have the maximum intensity),  $\lambda$  is the vacuum wavelength and  $d$  is the (variable) grating period. The grating equation above defines the steered order as the first order of the diffraction grating. The semantics here are somewhat different from microwave phased array theory in which it is by definition the zero order that is steered. Also note that due to the nature of diffractive devices, steering is in general not continuous, though techniques can be used to make the steering appear continuous. A major advantage of diffractive devices is that the addressable angles can be randomly accessed. An additional advantage of diffractive beamsteerers is the potential for two-dimensional steering using a single device. However, fabrication limitations restrict two-dimensional steering to small angles at this time. One-dimensional diffractive beamsteerers, like the refractive beamsteerers can be cascaded to steer in two dimensions.

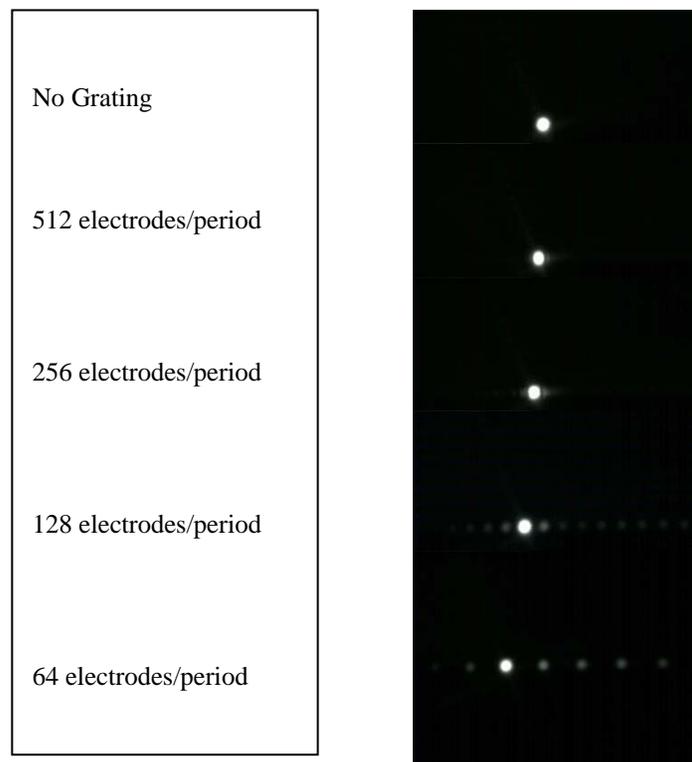
Both refractive and diffractive beamsteerers can be implemented using liquid crystal modulators. The phase shift induced by these modulators is achieved by a change in the optical path or by polarization modulation. Nematic and chiral smectic liquid crystal optical phase modulators are discussed below.

## **NEMATIC LIQUID CRYSTAL OPTICAL PHASE MODULATORS**

The most straightforward approach to constructing a phase grating using liquid crystal is to employ planar aligned nematic liquid crystal and tune the extraordinary index via an applied electric field. The polarization of the incident light is vibrating along the extraordinary axis of the liquid crystal and as the electric field is applied, the incident light encounters a varying optical path.

As an example of a nematic liquid crystal modulator applied to beamsteering consider the diffraction patterns of Figure 1. Here a series of voltage wedges of different periods has been written to a liquid crystal on silicon (VLSI-addressed) beam steering device. This is a one-

dimensional diffractive beamsteerer. The electrode elements are 1.0 micron wide lines separated by 0.8 micron wide spaces. There are 4096 individually addressable electrodes. The aperture is 7.4 mm x 6 mm. The design wavelength is 1.5  $\mu\text{m}$ , however, these devices are adaptable to other wavelengths, within the basic constraints of the backplane and liquid crystal material. An IR camera was used to capture the diffraction patterns. The angular deviation was restricted to  $\pm 3$  degrees for this device because the diffraction efficiency fell off to negligible levels for higher angles. This limitation is primarily a manifestation of the flyback region mentioned above. Improved liquid crystal materials (higher birefringence, sharper switching) and higher addressing voltages could double or triple the maximum useable angle. The optical damage threshold for these devices is determined by the absorption of the indium tin oxide (ITO) cover electrode. This is of course a wavelength dependent phenomenon but at 1.5 microns, damage to the ITO electrode occurs for laser fluences of approximately  $2 \text{ J/cm}^2$ . There are alternative materials for the transparent electrode (such as doped ZnO) that handle higher laser fluences.

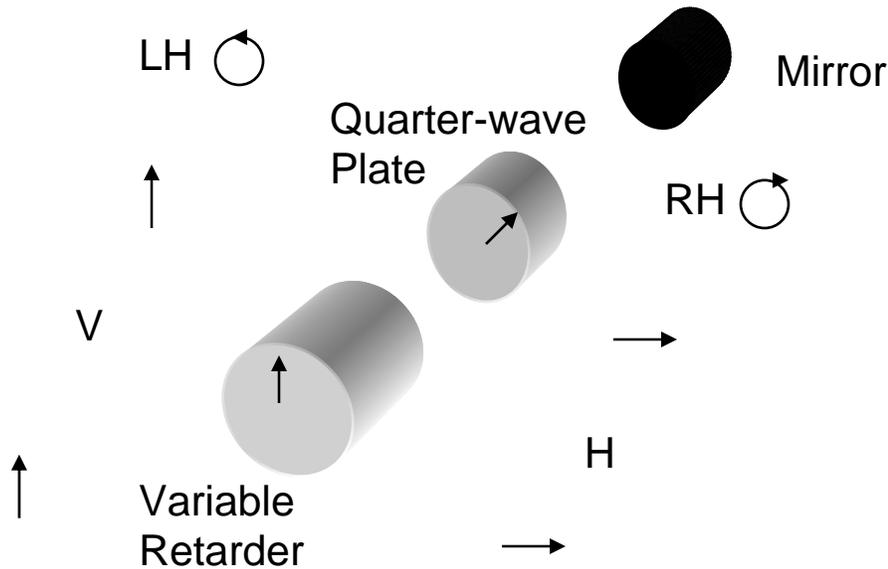


**Figure 1.** Diffraction patterns for voltage wedges applied to the 4096 electrode beamsteerer. For no grating, 87% of the incident linearly polarized 1.5 micron wavelength laser light is in the 0 order. After addressing, a majority of this light is deflected to the 1<sup>st</sup> order as defined by the grating equation above.

## Polarization independent implementation

One of the key limitations to liquid crystal phase modulators is that they require polarized light. This can be overcome using a polarization independent phase modulator [2]. The trade-off for polarization independent operation is an increase in the thickness of the modulating layer. This affects speed and large angle efficiency.

Figure 2. shows the structure for a polarization independent phase modulator. For the example shown here, vertically polarized light is modulated by the active liquid crystal waveplate. On reflection from the quarter-wave plate and mirror combination the light has rotated to the orthogonal state and what was horizontally polarized entering the modulator is now vertically polarized and it is modulated. The light that was modulated on the first pass is not modulated on reflection.



**Figure 2.** Schematic of polarization independent phase modulator. This forms a common path modulator. Path differences for the two polarizations are negligible for the thin liquid crystal layers (a few microns) commonly used in optical phased arrays with  $2\pi$  resets i.e. the operation occurs in the thin grating (Raman-Nath diffraction) regime.

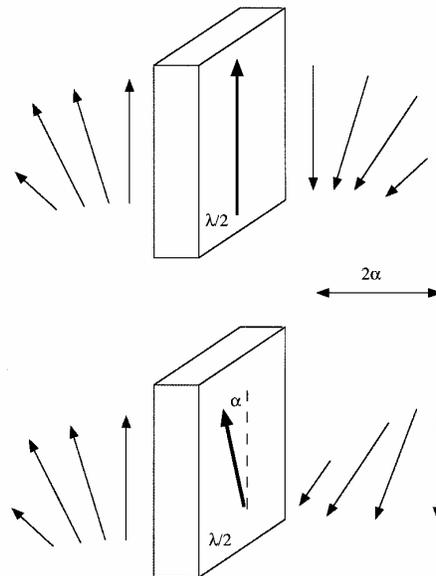
## PHASE MODULATION USING CHIRAL SMECTIC LIQUID CRYSTALS

Nematic liquid crystal beam deflectors operate on the principle of refractive index modulation. The novel approach using chiral smectic liquid crystals discussed here instead uses the topological phase shift produced by a rotative half-wave plate operating on a circularly polarized optical field. Mechanically actuated topological phase shifters were used on the first U. S. phased array radar during World War II [3]. The method for obtaining topological phase shift using rotative retarders far precedes the use in radar systems. In a 1941 paper developing a mathematical formalism for analyzing polarization systems, Jones presented examples of three-retarder structures [4]. One of these involved bounding a rotative half-wave retarder by quarter-wave retarders to form a variable phase shifter (QHQ). Furthermore, Jones referred to an 1859 paper by Billet, who asserted that these structures were known to Fresnel.

This topological phase shift technology is intrinsically suitable for applications in which circularly polarized laser light is used. However, it can be easily modified for use in systems employing linear or elliptical polarization. Moreover, due to the wavelength insensitive phase modulation scheme, the device can operate on multiple wavelengths.

### Optical phase shift using circular polarization modulation.

Figure 3 illustrates the principle of optical phase modulation using circular polarization. A circularly polarized incident optical field can be modeled as a sequence of instantaneous linear polarization states whose oscillation direction progresses in a circular manner as the optical field propagates. That is, the tip of the polarization vector (represented by the arrows in the figure)



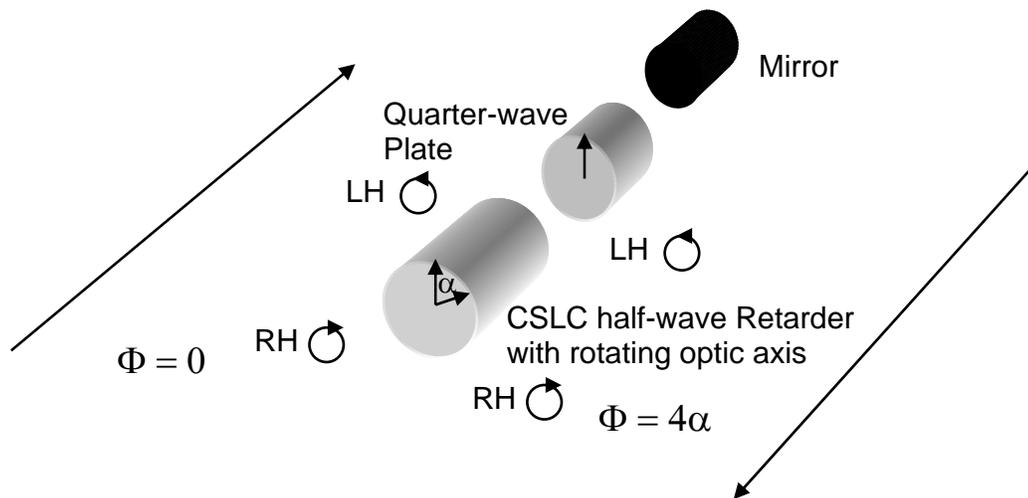
**Figure 3.** Optical Phase Shift induced on circularly polarized light by re-orientation of the optic axis of a half wave retarder.

will trace out a helix. Upon transmission through a half-wave retarder, these instantaneous linear states are reflected about the optic axis of the retarder, resulting in a handedness change for the circularly polarized field. If the optic axis of the half-wave retarder is reoriented by an angle  $\alpha$ , an additional relative phase of  $2\alpha$  is induced on the optical field. This relative induced phase is sometimes referred to as geometric phase.

Because the induced phase is the result of polarization modulation, it exhibits more broad band behavior than phase shift due to index change. With a planar aligned chiral smectic liquid crystal (CSLC) wave plate, the molecular tilt in the transverse plane changes as a function of applied electric field. This rotates the optic axis of the liquid crystal modulator. The position of the optic axis is the same for all wavelengths and ideally so is the induced phase shift. [5].

### Reflection-mode phase modulators

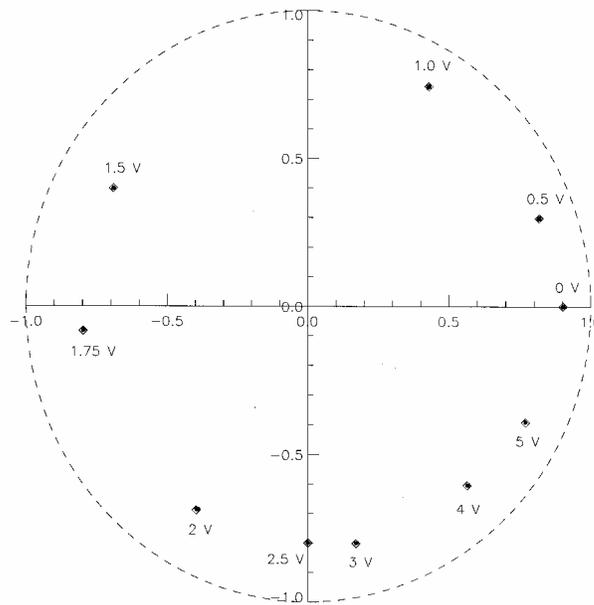
To implement a liquid crystal topological phase shifter, the rotative half-wave plate is ideally a device of fixed retardation and variable transverse orientation. For chiral smectic liquid crystals the maximum reorientation of the optic axis is  $90^\circ$ , and the maximum phase modulation depth is  $180^\circ$  for a transmission mode device. By using a passive quarter-wave retarder and mirror, twice the phase modulation depth can be achieved. The increase in phase modulation depth is due to the quarter-wave plate and mirror preserving the handedness of the reflected circularly polarized optical field. As seen in Figure 4, its structure is similar to the polarization independent modulator.



**Figure 4.** Schematic of chiral smectic liquid crystal phase modulator. The relative phase shift is 4 times the optic axis reorientation.

For an array of reflection-mode phase modulators, the active portion of the modulator should be situated as close to the reflecting surface as possible to limit the effects of diffraction between successive passes. Most means of obtaining a quarter-wave of retardation are too thick to be practical for use in efficient high resolution multiple pixel phase modulators. Three solutions have been identified: passivated form-birefringence quarter-wave retarders, polymer nematic liquid crystal quarter-wave retarders, or replacing the quarter-wave plate and mirror by a single element: a film of polymer cholesteric liquid crystal.

A bulk modulator based on the half-wave plate and cholesteric mirror geometry has previously been reported [6]. Since then, an integrated modulator has been implemented in which one of the substrates for the active half-wave retarder was a polymer cholesteric liquid crystal mirror. This mirror was on the order of  $8\ \mu\text{m}$  thick and the measured reflection was 96.5% of left hand circularly polarized light at 633 nm. The device was gapped using  $2.1\ \mu\text{m}$  spacers. It was determined that the chiral smectic liquid crystal cell was a half-wave retarder at 616 nm. For testing, the device was placed in a Michelson interferometer and a variable amplitude square-wave voltage of 0.3 Hz was applied. Using the phase modulation depth obtained from fringe shift and the square root of the normalized optical intensity measured by a power meter, the complex amplitude response of the modulator has been obtained. The complex plane representation of the integrated half-wave cholesteric phase modulator for a wavelength of 632.8 nm is shown in Figure 5. Here an ideal  $360^\circ$  lossless phase modulator would correspond to the unit circle. The loci of points that characterize the integrated modulator trace out a circle with its center shifted along the positive real axis. The radius is less than one because of losses, and the center is shifted along the real axis because the modulator performance is better in the first and fourth quadrants i.e. for small and large optic axis orientations



**Figure 5.** Complex amplitude response of a rotative phase shifter in the half-wave and cholesteric mirror geometry.

The complex amplitude response for the single pixel device shown above indicates that these phase shifters are excellent candidates for implementation in a phased array geometry. However, for high resolution, two-dimensional phased arrays, deposition of an electrode structure onto the polymer cholesteric mirror becomes problematic. If the electrode structure is implemented beneath the polymer cholesteric liquid crystal mirror, the voltage levels necessary for addressing become prohibitive [7]. The solution is to use newly developed polymer nematic retarders. For these polymer nematic retarders, a 2-3 micron thick layer can produce a quarter-wave retardation in the near IR. Since the polymer nematic layer is much thinner than the equivalent polymer cholesteric layer, the active portion of the modulator can be addressed through the polymer quarter-wave retarder without a prohibitive voltage drop.

### **Rotative half-wave plates based on analog switching chiral smectic liquid crystal**

The liquid crystal rotative half-wave retarders which comprise the active portion of the modulators must exhibit analog reorientation of the optic axis in the plane transverse to the direction of propagation of the incident optical field. Analog transverse optic axis re-orientation involves using the device in an uncommon configuration. Generally chiral smectic liquid crystals are used in a binary mode [8]. However, this is only a special case of the switching possibilities exhibited by these materials.

Chiral smectic liquid crystals (CSLC) can be categorized according to whether the spontaneous polarization is induced by an applied field (electroclinic), or if it is intrinsically present (ferroelectric). Electroclinic liquid crystal devices exhibit rapid (microseconds) response with soft-mode (transverse plane) switching, however, most electroclinic materials developed to date do not exhibit satisfactory modulation depth. Moreover, the electro-optic properties of electroclinic devices are highly temperature dependent.

The devices discussed here have only a single stable director orientation in the absence of applied electric field. For the phase modulation application considered here, it is desirable that the monostable state is at the extreme of the cone. That is, the projection of the director in the transverse plane with respect to the smectic layer normal is equal to the tilt angle. Others have reported uniform switching based on reorientation of the director from a quiescent state upon application of electric field to thin FLC cells exhibiting surface suppression of the  $\text{SmC}^*$  helix. However, their devices were either bistable or monostable with the quiescent state near the center of the cone, such that the optic axis was approximately normal to the smectic layers .

The advantage of the devices discussed here which exhibit uniform switching and which have the stable state at an angle equal to the tilt angle relative to the smectic layer normal are:

(1) The monostable state has uniform director orientation. A device with a stable state at an angle other than the tilt angle with respect to the smectic layer normal must have a twisted director orientation, and will consequently exhibit some optical activity.

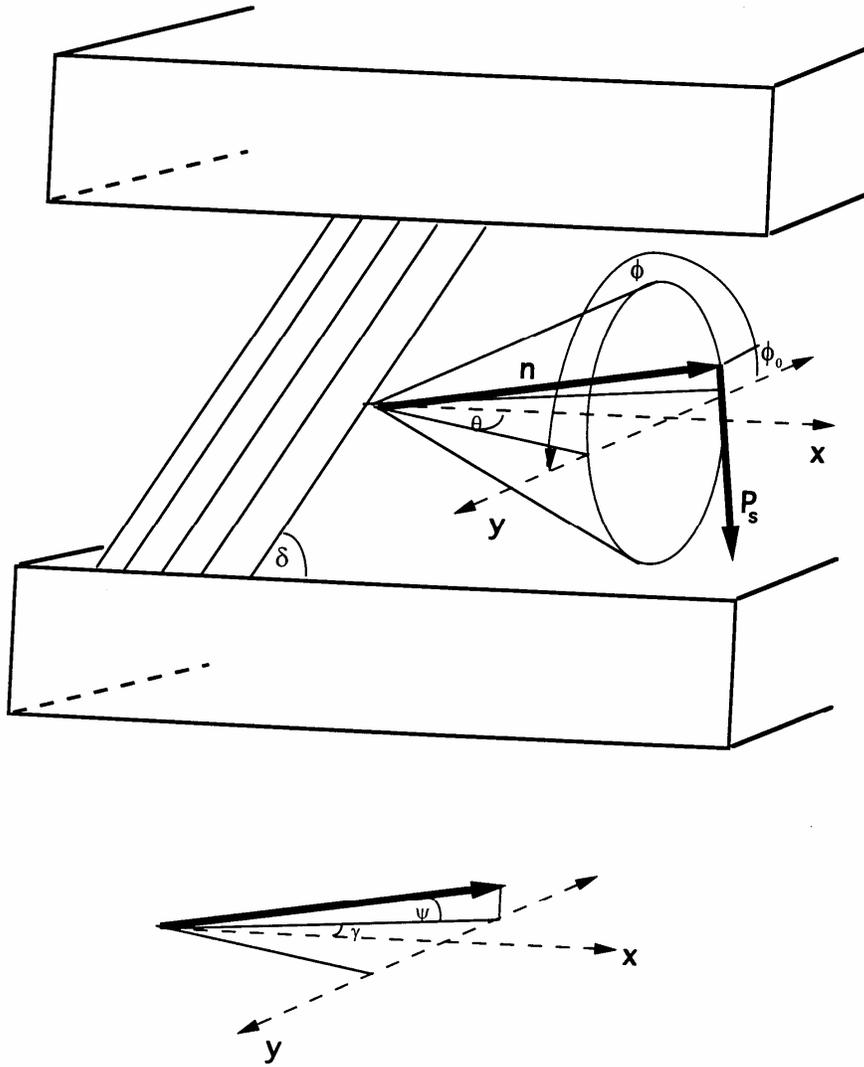
(2) Since the monostable state is already at the extreme of the chiral smectic tilt angle cone, the directors can relax (or be driven) back to the monostable state without the need for a complex

driving scheme to produce a consistent quiescent state. That is, the devices ideally do not exhibit hysteresis for the zero field state.

This second point needs further emphasis. If the monostable state occurs at an angle other than the tilt angle, a special driving scheme needs to be used to mitigate hysteresis. Perhaps the simplest solution to this problem, is to drive the devices at the frequency at which the hysteresis loop disappears. The catch is, this frequency may be too low. For example, measurements of the material BDH-764E after transition into the SmC\* phase demonstrate a minimum in hysteresis for a driving frequency of 250 Hz. It is evident that this material could not be used to implement a high quality sub-millisecond analog FLC device. To come up with a material that exhibits a minimum of hysteresis at 1 kHz for a zero-field state that is not on the cone angle extreme would require an extensive materials search, if not an all-out chemical synthesis effort. For this reason, analog FLC devices that operate such that the monostable state is at the cone-angle extreme are the best approach for implementing optical phase modulators using topological phase shift.

Figure 6 shows the general director geometry for the devices considered here. A thin chiral smectic liquid crystal (CSLC) layer is sandwiched between two substrates. The CSLC molecules are arranged in layers, and within the layers the molecules are oriented with respect to the smectic layer normal at the cone angle  $\theta$ . The layers are generally either tilted or form chevrons at an angle  $\delta$  with respect to the substrate. In addition, the molecules themselves may be pretilted at an angle with respect to the substrate such that the stable state is  $\phi = \phi_0$ . The molecular director,  $n$ , defines the slow axis. The spontaneous polarization is normal to the director, and directed into (or out of) the substrate in the zero-field and maximum field states.

Upon application of an electric field, the spontaneous polarization couples to the field and the director moves along the cone defined by the tilt angle  $\theta$  and having an axis  $x$  normal to the smectic layers. So the azimuth angle  $\phi$  represents the orientation of the liquid crystal director on the surface of the tilt angle cone. Under the influence of applied electric field, the director orientation,  $\phi$ , varies continuously from  $\phi = \phi_0$  (at or near  $\phi = 0^\circ$ ) up to an extreme of  $\phi = 180^\circ$ . Thus, the projection of the directors onto the substrates which represents the effective optic axis  $\gamma$ , continuously rotates from  $\gamma = \theta$  to  $\gamma = -\theta$ . The switching occurs more readily for molecules near the center of the device such that at intermediate applied fields a small twist develops. That is, the molecules near the surface slightly lag in switching about the cone due to surface anchoring forces. For low applied electric fields, all of the molecules are near the quiescent state. The reorientation of the optic axis is small and the director conformation is relatively uniform throughout the device thickness. As the field is increased the molecules in the center of the cell proceed further about the cone than those near the surfaces resulting in a slight twist in the director orientation throughout the device thickness. For large applied fields, essentially all of the molecules are switched to the opposite cone extreme, resulting again in a uniform director conformation.



**Figure 6.** Schematic of CSLC retarder geometry. The molecules are arranged in layers and oriented uniformly at the cone angle  $\theta$  with respect to the layer normal. The layers are tilted at an angle  $\delta$  with respect to the substrate.  $\phi$  is the azimuthal cone angle, and  $\phi_0$  is the molecular pretilt. The liquid crystal director  $\mathbf{n}$  defines the slow axis, which is oriented at an angle  $\psi$  with respect to the substrate. The spontaneous polarization,  $\mathbf{P}_s$ , is normal to the director. The effective optic axis in the plane of the substrates is  $\gamma$ .

Consider first the situation where the smectic layers are normal to the substrates ( $\delta = 0^\circ$ ) and the molecules have no pretilt ( $\phi_0 = 0^\circ$ ). This is sometimes referred to as the so-called bookshelf geometry [9]. Here the monostable state is defined as  $\phi = 0^\circ$ ,  $\gamma = \theta$ . Upon application of an electric field, the spontaneous polarization couples such that the director moves along the surface of the cone, and  $\phi$  increases continuously depending on the electric field strength. For large enough fields, the directors rotate about the cone up to the extreme position of the azimuthal cone angle  $\phi = 180^\circ$ . In this orientation,  $\gamma = -\theta$  such that the spontaneous polarization

is now pointing in the direction of the electric field essentially throughout the cell. The effective optic axis orientation,  $\gamma$ , has undergone an analog rotation in the plane of the substrates up to a maximum rotation of  $2\theta$ . If the electric field is removed, the director *relaxes* back to the original monostable state:  $\phi = 0^\circ$  and  $\gamma = \theta$ . If from the extreme switched state ( $\phi = 180^\circ$ ,  $\gamma = -\theta$ ), the sign of the electric field is reversed, the spontaneous polarization couples to the field and *drives* the molecular director back along the cone such that  $\phi$  decreases to  $\phi = 0^\circ$  and  $\gamma$  continuously rotates from a value of  $-\theta$  to  $\theta$ .

The dielectric anisotropy,  $\Delta\epsilon$ , will also couple to the electric field. This torque is weaker than the polarization coupling and is second order. Depending on the sign, the dielectric anisotropy will contribute or detract from the director re-orientation [10]. The torque due to spontaneous polarization is proportional to  $P_s \cdot E$ . The torque due to dielectric anisotropy is proportional to  $\Delta\epsilon E^2$ . Due to the high spontaneous polarization for this material, the polarization torque dominates and at fields on the order of  $10\text{V}/\mu\text{m}$  the dielectric torque (based on a  $\Delta\epsilon$  of 2 which is typical for CSLC materials) will contribute less than 8% of the total torque acting to reorient the directors. It should be noted that in previous experiments on CSLC retarders complete switching took place for fields between 2.5 and  $10\text{V}/\mu\text{m}$ . Consequently, contributions to switching due to dielectric anisotropy are secondary for these devices.

One important contribution that may occur for a material with negative dielectric anisotropy is that the dielectric anisotropy helps to limit the out of plane tilt of the optic axis. As the molecules are switched about the tilt cone by coupling to the spontaneous polarization, a negative dielectric coefficient will act to keep the molecules confined to the transverse plane. The net effect of this is that the out of plane tilt (and consequently the retardance change that accompanies director reorientation discussed below) is reduced, allowing the device to more closely resemble the desired configuration of a fixed retarder with variable orientation.

During uniform (or quasi-uniform) switching, a normally incident optical field will encounter a change in the retardation of a device with liquid crystals in the bookshelf geometry. The retardation decreases as the director moves along the cone from the original monostable state ( $\phi = 0^\circ$ ,  $\gamma = \theta$ ) to ( $\phi = 90^\circ$ ,  $\gamma = 0^\circ$ ). Further reorientation along the cone results in an increase in retardation as the optic axis moves from ( $\phi = 90^\circ$ ,  $\gamma = 0^\circ$ ) to the extreme induced orientation ( $\phi = 180^\circ$ ,  $\gamma = -\theta$ ). This is a result of the change in the projection of the extraordinary index as the molecules rotate about the cone. In addition to the possibility of contributions from negative dielectric anisotropy as mentioned above, a tilted (or chevron) layer structure can also reduce the out of plane tilt perceived by the incident optical field.

The behavior of these CSLC retarders switching under the influence of an applied electric field suggests the following process. The initial (zero field) state is uniform, with the optic axis at an angle equal to the tilt angle with respect to the smectic layer normal. This state is defined by the angles  $\gamma = \theta$ ,  $\phi = 0^\circ$ . Now,  $\alpha$  is the optic axis rotation from this quiescent state. (In the geometry of Figure 6,  $\alpha = \theta - \gamma$ .) Upon application of an electric field,  $\alpha$  increases up to a maximum rotation of  $88^\circ$ . Meanwhile, the azimuthal angle increases up to a maximum  $180^\circ$  as the director proceeds about the tilt angle cone. The change in these angular quantities in addition to the changing layer tilt affect the birefringence and hence the optical retardation. Furthermore,

the twist that develops due to the molecules in the bulk switching more readily than those near the surface also changes during switching. In the quiescent state the director distribution is uniform. When an electric field is initially applied to the device, a slight twist develops, then as more of the molecules rotate under influence of the field, the twist decreases such that the molecular conformation is again essentially uniform. This description for switching is corroborated by experimental observations[11].

## CONCLUSION

An optical phased array is conveniently implemented using a liquid crystal writeable grating. Ideally the phase profile produced by the grating emulates a linear ramp using a periodic saw-tooth function having a  $2\pi$  ( $360^\circ$ ) modulation depth. Different modulating structures can allow polarization independent operation or use of analog switching ferroelectric liquid crystal devices.

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