Extinction ratio measurements on high purity linear polarizers

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ABSTRACT

Measured extinction ratio on high quality linear polarizers depends on test system geometry. The measurement becomes especially challenging for polarizers with extinction ratio expected to exceed 10^6. We describe methods capable of measurements of high purity polarizers at and above 10^6 extinction ratio. We discuss the geometrical factors affecting the measured results that may be pertinent in determining what performance is achievable in a users system. We describe methods for computing extinction ratios without an absolute reference perfect polarizer with infinite extinction ratio and for rank ordering performance of a set of several polarizers. Measurement results are presented for several high performance polarizers including Glan-Thompson polarizers, dichroic glass polarizers and Meadowlark Optics Ultra Broadband Polarizers.

Keywords: polarization, birefringence

1. INTRODUCTION

Jones and Hurwitz [1,2] have shown in 1941 that "for light of a given wavelength, an optical system containing any number of partial polarizers and rotators is optically equivalent to a system containing only two elements - one a partial polarizer, and the other a rotator." [2] Moreover, if the transmission axes of all partial polarizers in the system are aligned along the x axis or the y axis, the system is equivalent to only one partial polarizer. Since we employ unpolarized light to measure the transmission intensities of a polarizer system, we are using Mueller calculus and not Jones calculus. [3–5]. We provide descriptions and error estimates for new methods to determine extinction ratios of high-purity polarizers, and discuss factors in the geometrical arrangement of the polarizers and limitations of the measurement instrumentation.

2. MATHEMATICAL PRELIMINARIES

A partial polarizer with transmission axes aligned along the x axis and y axis is represented by the Mueller matrix of the form

$$M\left(p_x, p_y\right) = \frac{1}{2}\begin{pmatrix} p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\ p_x^2 - p_y^2 & p_x^2 + p_y^2 & 0 & 0 \\ 0 & 0 & 2p_xp_y & 0 \\ 0 & 0 & 0 & 2p_xp_y \end{pmatrix}, \tag{1}$$

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where the p_x and p_y are the (positive) amplitude attenuation coefficients along the x and y axis, respectively [3]. For $p_x \geq p_y$ the contrast ratio (or extinction ratio) is defined as

$$R = p_x^2/p_y^2 \; . \tag{2}$$

The contrast ratio is always at least 1 according to this definition, and equals 1 if and only if $p_x = p_y$, which describes a neutral density filter.

The combination of two or more partial polarizers with their transmission axes oriented either along the x and y axis is represented by the matrix product of their Mueller matrices, where the matrices are arranged from right to left in the order in which a normally incident ray encounters them. The combination of a partial polarizer with amplitude attenuation coefficients p_{1x} and p_{1y} and a second one with coefficients p_{2x} and p_{2y} along the x and y axis, respectively, is then found, after a little algebraic reduction, to be described by the matrix of a single partial polarizer, whose attenuation coefficients are the product of the respective attenuation coefficients of the individual polarizers, both along the x and the y axis:

$$\begin{aligned} & M\left(p_{1x}, p_{1y}\right) \cdot M\left(p_{2x}, p_{2y}\right) \\ = & \frac{1}{2}\begin{pmatrix} p_x^2+p_y^2 & p_x^2-p_y^2 & 0 & 0 \\ p_x^2-p_y^2 & p_x^2+p_y^2 & 0 & 0 \\ 0 & 0 & 2p_xp_y & 0 \\ 0 & 0 & 0 & 2p_xp_y \end{pmatrix} \cdot \\ & \frac{1}{2}\begin{pmatrix} p_x^2+p_y^2 & p_x^2-p_y^2 & 0 & 0 \\ p_x^2-p_y^2 & p_x^2+p_y^2 & 0 & 0 \\ 0 & 0 & 2p_xp_y & 0 \\ 0 & 0 & 0 & 2p_xp_y \end{pmatrix} \\ = & M\left(p_{1x}p_{2x}, p_{1y}p_{2y}\right) . \end{aligned} \tag{3}$$

Since $p_{1x}p_{2x} = p_{2x}p_{1x}$ and $p_{1y}p_{2y} = p_{2y}p_{1y}$, Eq. (3) implies that the combination of two aligned polarizers is commutative, i.e.

$$M\left(p_{1x}, p_{1y}\right) \cdot M\left(p_{2x}, p_{2y}\right) = M\left(p_{2x}, p_{2y}\right) \cdot M\left(p_{1x}, p_{1y}\right) . \tag{4}$$

By mathematical induction both results (3) and (4) can be generalized to a stack of an arbitrary number N of partial polarizers with their transmission axes aligned along the x and y axes, i.e.

$$\prod_{i=1}^{N} M\left(p_{ix}, p_{iy}\right) = M\left(\prod_{i=1}^{N} p_{ix}, \; \prod_{i=1}^{N} p_{iy}\right) , \tag{5}$$

and the matrix product is invariant under changes of the order of the factors, which corresponds to the geometrical order of the polarizers in the combination stack.

3. PAIR CONTRAST RATIO OF TWO IMPERFECT POLARIZERS: DEFINITION AND CALCULATION

If unpolarized light is transmitted with normal incidence through a pair of polarizers with contrast ratios R_1 and R_2, their pair contrast ratio, defined as the ratio r_{12} of transmission intensities for parallel alignment and for crossed alignment of the polarizers, is related to R_1 and R_2 by the formula,

$$r_{12} = \frac{R_1R_2+1}{R_1+R_2}. \tag{6}$$

Proof. Let $p1a$ and p_{1b} be the attenuation coefficients of the first polarizer (ordered such that $p_{1b} \leq p_{1a}$), and let p_{2a} and p_{2b} be the attenuation coefficients of the second polarizer (ordered such that $p_{2b} \leq p_{2a}$). Then the contrast ratios of the polarizers are

$$R_1 = p_{1a}^2/p_{1b}^2, \; R_2 = p_{2a}^2/p_{2b}^2. \tag{7}$$

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The unpolarized transmission intensity for parallel polarizers is

$$\begin{aligned} I_{\parallel} &= \left[M\left(p_{1a}, p_{1b}\right) \cdot M\left(p_{2a}, p_{2b}\right) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right]_1 \\ &= \left[M\left(p_{1a}, p_{1b}\right) \cdot M\left(p_{2a}, p_{2b}\right) \right]_{11} \\ &= \left[M\left(p_{1a}p_{2a}, p_{1b}p_{2b}\right) \right]_{11} \\ &= \frac{1}{2}\left(p_{1a}^2 p_{2a}^2 + p_{1b}^2 p_{2b}^2\right) \ , \end{aligned} \tag{8}$$

and, similarly, the unpolarized transmission intensity for crossed polarizers,

$$\begin{aligned} I_{\perp} &= \left[M\left(p_{1a}, p_{1b}\right) \cdot M\left(p_{2b}, p_{2a}\right) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right]_1 \\ &= \left[M\left(p_{1a}, p_{1b}\right) \cdot M\left(p_{2b}, p_{2a}\right) \right]_{11} \\ &= \left[M\left(p_{1a}p_{2b}, p_{1b}p_{2a}\right) \right]_{11} \\ &= \frac{1}{2}\left(p_{1a}^2 p_{2b}^2 + p_{1b}^2 p_{2a}^2\right) \ . \end{aligned} \tag{9}$$

Hence the pair contrast ratio as defined above is

$$r_{12} = \frac{I_{\parallel}}{I_{\perp}} = \frac{\frac{1}{2}\left(p_{1a}^2 p_{2a}^2 + p_{1b}^2 p_{2b}^2\right)}{\frac{1}{2}\left(p_{1a}^2 p_{2b}^2 + p_{1b}^2 p_{2a}^2\right)} = \frac{\dfrac{p_{1a}^2}{p_{1b}^2} \cdot \dfrac{p_{2a}^2}{p_{2b}^2} + 1}{\dfrac{p_{1a}^2}{p_{1b}^2} + \dfrac{p_{2a}^2}{p_{2b}^2}} = \frac{R_1 R_2 + 1}{R_1 + R_2} \tag{10}$$

□

4. PROPERTIES OF THE PAIR CONTRAST RATIO

From formula (6) we derive some properties of the relationship between the pair contrast ratio (which can be determined by measuring unpolarized transmission intensities) and the intrinsic contrast ratios of the two polarizers. These properties are illustrated in the model graph in Fig. 1 and Table 1 below.

4.1 Approximation for $R_1, R_2 \gg 1$

For $R_1, R_2 \gg 1$ formula (6) simplifies to

$$r_{12} \approx \frac{R_1 R_2}{R_1 + R_2}, \tag{11}$$

which is reminiscent of the formula for the resistance of a combination of two parallel resistors in an electronic circuit, and can equivalently be written as,

$$\frac{1}{r_{12}} \approx \frac{1}{R_1} + \frac{1}{R_2}. \tag{12}$$

Just as the resistance of two parallel resistors cannot exceed either one of the two resistors [6], the pair contrast ratio cannot exceed either of the contrast ratios R_1 and R_2, as will be formally shown below in subsection 4.4 for the exact relation (6) .

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4.2 Monotonicity

For fixed $R_1 > 1$ the pair contrast ratio as function of R_2 (and similarly for fixed $R_2 > 1$ as function of R_1) is strictly increasing.

Proof. Taking partial derivatives in Eq. (6) implies for all $R_1, R_2 > 1$ that

$$\frac{\partial r_{12}}{\partial R_2} = \frac{R_1^2 - 1}{(R_1 + R_2)^2} > 0, \tag{13}$$

$$\frac{\partial r_{12}}{\partial R_1} = \frac{R_2^2 - 1}{(R_1 + R_2)^2} > 0, \tag{14}$$

from which the assertion follows. ∎

4.3 Lower bound

The minimum of r_{12} has the value 1; and $r_{12} = 1$ if and only if $R_1 = 1$ or $R_2 = 1$.

Proof. By substitution into Eq. (6) we see that $R_1 = 1$ or $R_2 = 1$ implies $r_{12} = 1$. Conversely, let $r_{12} = 1$, i. e.

$$\frac{R_1 R_2 + 1}{R_1 + R_2} = 1. \tag{15}$$

Multiplying by $(R_1 + R_2)$ and rearranging terms yields

$$(R_1 - 1) \cdot (R_2 - 1) = 0, \tag{16}$$

which implies that $R_1 = 1$ or $R_2 = 1$.

It remains to be shown that $r_{12} \geq 1$ for all $R_1, R_2 \geq 1$. Since

$$(R_1 - 1) \cdot (R_2 - 1) \geq 0, \tag{17}$$

we obtain, after rearranging tems,

$$\frac{R_1 R_2 + 1}{R_1 + R_2} \geq 1. \tag{18}$$

∎

4.4 Upper bound

The pair contrast ratio is bounded above by the minimum of R_1 and R_2.

Proof. Because of $R_1 \geq 1$, Eq. (6) implies that

$$r_{12} = \frac{R_1 R_2 + 1}{R_1 + R_2} \leq \frac{R_1 R_2 + R_1}{1 + R_2} = R_1, \tag{19}$$

and in the same way that

$$r_{12} = \frac{R_1 R_2 + 1}{R_1 + R_2} \leq \frac{R_1 R_2 + R_2}{R_1 + 1} = R_2. \tag{20}$$

From (19) and (20) it follows that $r_{12} \leq \min(R_1, R_2)$. ∎

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5. APPLICATIONS TO DETERMINE OR RANK UNKNOWN CONTRAST RATIOS

5.1 Determine one unknown contrast ratio

Since the unpolarized intensities of light normally incident on a pair of polarizers can be directly measured with a detector for both parallel and crossed polarizers, formula (6) enables us to solve for an unknown contrast ratio of one of the two polarizers, if the contrast ratio of the other one is already known. Therefore if, e.g., R_1 is known and $r_{12} = I_{\|}/I_{\perp}$ is determined by measuring $I_{\|}$ and $I_{\perp}$, then the unknown contrast ratio of the second polarizer is

$$R_2 = \frac{r_{12}\ R_1 - 1}{R_1 - r_{12}}. \tag{21}$$

Assuming $R_1 \gg 1$ and $R_2 \gg 1$, we have the approximation

$$R_2 \approx \frac{r_{12}\ R_1}{R_1 - r_{12}}. \tag{22}$$

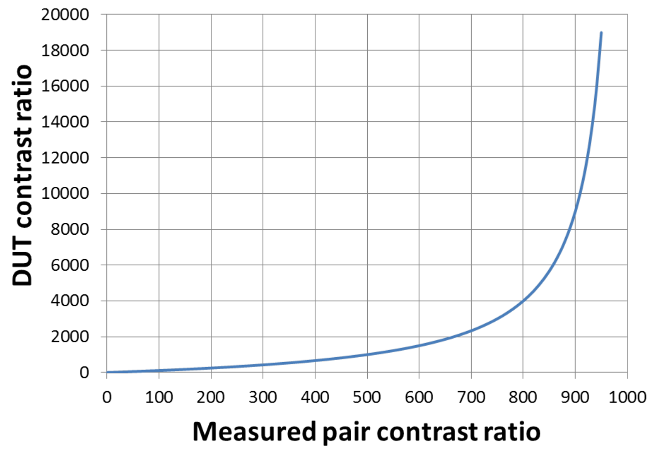

Figure 1: For a contrast ratio of 1000 (chosen as example) of the reference polarizer, the contrast ratio of the DUT polarizer can be calculated from the measured pair contrast (contrast) ratio of the intensities of unpolarized light through the parallel and crossed combinations of the DUT and the reference polarizer. The steeper the slope (for higher DUT CRs), the greater the relative error in this method. We recommend to use this method for DUT CRs up to 400 % of the reference CR, corresponding to a measured pair contrast ratio of 80 % of the reference CR.

Table 1: Some tabulated value pairs of the DUT CR vs. pair CR graph in Fig. 1, both as absolute quantities and as a percentage of the reference CR of 1000.

1	1 (i.e. ideal neutral density filter)
100 = 10%	111 = 11.1%
340=34%	515 = 51.5%
500=50%	1000 = 100%
800=80%	4000 = 400%
900=90%	9000 = 900%
1000=100%	∞ (i.e. ideal polarizer)

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5.2 Rank two unknown contrast ratios

For three polarizers with contrast ratios $R_1 > 1$, R_2 and R_3 we have the ranking theorem that $r_{12} < r_{13}$ implies $R_2 < R_3$.

Proof. $r_{12} < r_{13}$ means that

$$\frac{R_1 R_2 + 1}{R_1 + R_2} < \frac{R_1 R_3 + 1}{R_1 + R_3}. \tag{23}$$

Multiplying by denominators (both positive), subtracting common terms and factoring out $\left(R_1^2 - 1\right)$ yields

$$R_2 \left(R_1^2 - 1\right) < R_3 \left(R_1^2 - 1\right). \tag{24}$$

By assumption $R_1 > 1$, so dividing the inequality by the common positive factor $\left(R_1^2 - 1\right)$ yields the assertion $R_2 < R_3$. ☐

5.3 Determine three unknown contrast ratios

For three polarizers with unknown contrast ratios R_1, R_2 and R_3 we can measure the unpolarized intensities $I_{ij\parallel}$ and $I_{ij\perp}$ (where $i, j = 1, 2, 3$ and $i \neq j$) for each of three pairs of these polarizers for parallel and crossed mode, determine the corresponding pair contrast ratios $r_{ij} = I_{ij\parallel}/I_{ij\perp}$ and apply the approximation in Eq. (12) to obtain a linear system of three equations for the reciprocals of the three unknown contrast ratios:

$$\begin{aligned} \frac{1}{r_{12}} &\approx \frac{1}{R_1} + \frac{1}{R_2}, \\ \frac{1}{r_{23}} &\approx \frac{1}{R_2} + \frac{1}{R_3}, \\ \frac{1}{r_{13}} &\approx \frac{1}{R_1} + \frac{1}{R_3}. \end{aligned} \tag{25}$$

The solutions are easily obtained by adding any two of these equations, subtracting the third and dividing by 2:

$$\begin{aligned} \frac{1}{R_1} &\approx \frac{1}{2}\left(\frac{1}{r_{12}} + \frac{1}{r_{23}} - \frac{1}{r_{13}}\right), \\ \frac{1}{R_2} &\approx \frac{1}{2}\left(-\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{13}}\right), \\ \frac{1}{R_3} &\approx \frac{1}{2}\left(\frac{1}{r_{12}} - \frac{1}{r_{23}} + \frac{1}{r_{13}}\right). \end{aligned} \tag{26}$$

Remark 1: The matrix of the linear system in Eq. (25) for the reciprocals $(1/R_1)$, $(1/R_2)$ and $(1/R_3)$ of the unknown contrast ratios, i.e. the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \tag{27}$$

commutes with its conjugate transpose (i. e. $A^* A = A A^*$) and has eigenvalues 2, $0.5\left(1 + i\sqrt{3}\right)$ and $0.5\left(1 - i\sqrt{3}\right)$, hence it is a hermitian matrix, and its condition number is the ratio of the greatest and the smallest of the respective absolute values 2, 1 and 1 of these eigenvalues. This condition number is 2, which indicates a well-conditioned matrix [7] and therefore a well-conditioned linear system in Eq. (25), whose solutions (as explicitly given in Eq. (26)) do not sensitively depend on the components of the right-hand side vector $\left(1/r_{12}, 1/r_{23}, 1/r_{13}\right)^\top$.

Remark 2: For polarizers with small contrast ratios the set of approximate solutions (26) can serve as initial guess, if - in order to obtain a more exact solution - we subsequently find a numerical solution of the nonlinear system based on Eq. (6) for each of the three polarizer pairs using, e.g., Newton's method.

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6. ERROR ESTIMATES FOR PAIR MEASUREMENTS

If for a pair of polarizers the contrast ratio of one of them is bounded by two constants, i. e. $1 < A \le R_1 \le B$, then the resulting relative error ϵ for the contrast ratio R_2 of the other polarizer is bounded by

$$\epsilon \le \frac{(B-A)r_{12}}{2AB-(A+B)r_{12}} \,. \tag{28}$$

Proof. By Eq. (21), and using $r_{12} \ge 0$, we have

$$\frac{\partial R_2}{\partial R_1} = \frac{1-r_{12}^2}{(R_1-r_{12})^2} \le 0, \tag{29}$$

wherefore R_2, using the approximation in Eq. (22), is bounded by

$$C := \frac{r_{12}B}{B-r_{12}} \le R_2 \le \frac{r_{12}A}{A-r_{12}} =: D. \tag{30}$$

Choosing $R_2 := (C+D)/2$, i. e. as the average of its lower and upper bound, the relative error ϵ of R_2, is bounded by

$$\epsilon \le \frac{(D-C)/2}{R_2} = \frac{D-C}{C+D} = \frac{(B-A)r_{12}}{2AB-(A+B)r_{12}} \,. \tag{31}$$

$\square$

See Fig. 3 for an example of approximating the DUT contrast ratio, if the CR of the reference polarizer is known within an upper and lower bound.

Remark 1: For $B \to \infty$, i. e. only a lower bound for contrast ratio R_1 is known, the estimate in Eq. (28) reduces to

$$\epsilon \le \frac{r_{12}}{2A-r_{12}} \,. \tag{32}$$

See Fig. 4 for an example of approximating the DUT contrast ratio, if only a lower bound for the CR of the refence polarizer is known.

Remark 2: In Fig. 2 we illustrate the relative error of the calculated DUT polarizer CR as a function of the measured pair contrast ratio (or pair CR) r_{12}. If the pair CR is 50% of the reference CR, then the relative error of the calculated DUT CR is equal to the relative error of the reference CR. The estimate in Eq. (28) can be used to determine which contrast ratio of the reference polarizer is needed to achieve a desired relative error for the unknown contrast ratio of the polarizer under test.

7. SUMMARY

Using Mueller matrix calculus we showed how the ratio of measured unpolarized intensities of a pair of partial polarizers with parallel and with crossed transmission axes is related to the contrast (or extinction) ratios of the two polarizers by a simple formula.

We term this ratio of unpolarized intensities the pair contrast ratio of the two polarizers and show how it can be used to either solve for the unknown polarizer contrast ratio if the contrast ratio of the other polarizer is known, or, alternatively, to solve for three unknown polarizer contrast ratios, if their pair contrast ratio is determined from intensity measurements for each of the three possible pairs.

For the first method we supply an error estimate for the unknown contrast ratio of the polarizer under test in the case that the contrast ratio of the reference polarizer is not exactly known, but known within a range. For the alternative method, measuring pair contrast ratios of the three possible pairs of three similar polarizers, we show that the resulting linear system is well-conditioned, and provide an explicit approximate solution.

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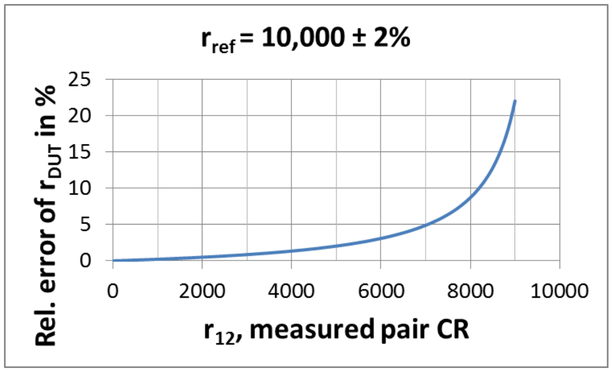

Figure 2: The relative error of the calculated DUT CR increases with higher purity of the polarizer under test. In this example we assume that the reference polarizer CR is 10,000 with a tolerance of 2%.

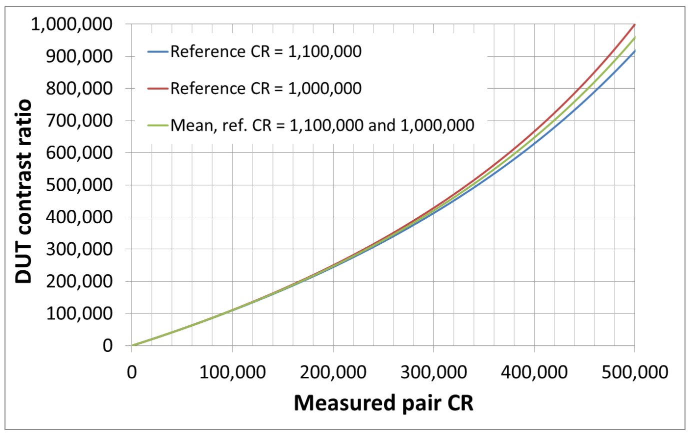

Figure 3: If an upper and a lower bound for the contrast ratio of the reference polarizer are known, an approximation for the contrast ratio of the polarizer under test is the arithmetic mean of calculated DUT CRs for the upper and lower bound of the reference CR.

APPENDIX A. EXAMPLES OF COMMERCIALLY AVAILABLE HIGH-QUALITY POLARIZERS

1. Glan Thompson Polarizer: The Glan Thompson polarizer is one of a class of polarizers using a birefringent crystal to separate the o and e (ordinary and extraordinary) rays into beams with different directions. Only the e ray is passed in the Glan Thompson polarizer with the o ray being rejected by total internal reflection. This eray is passed without deflection. Calcite is the much preferred crystal for these polarizers because the high birefringence of this crystal minimizes the polarizer path length and maximizes the angular field of performance. For many years this polarizer has been the preferred choice when excellent linear polarization purity or extinction ratio greater than 10^6 is required. However, the cost of these polarizers becomes very high for clear apertures exceeding 25 mm and is unavailable generally in apertures exceeding about 40 mm at any price. Also, the angular field of performance is less than that for plate or sheet polarizers.

2. Dichroic Glass Polarizer: This is one of a class of plate polarizers composed of a glass matrix imbedded aligned array of microscopic elongated metallic whiskers. The parallel linear alignment of theses metallic whiskers functions in a way similar to a wire grid polarizer. The extinction ratio is easily exceeds 10^6, but usually over a wavelength range of 700 nm or less.

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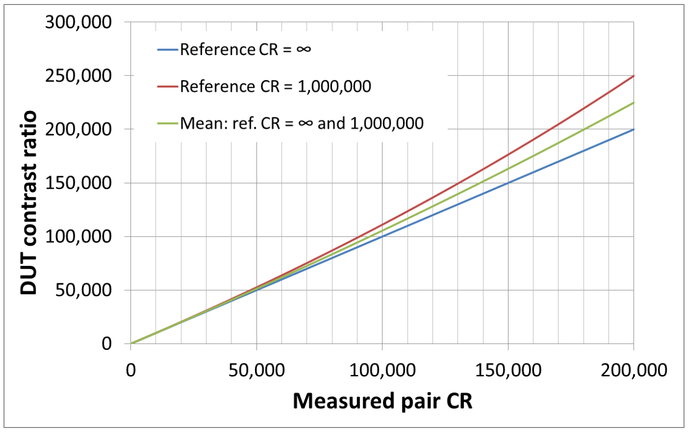

Figure 4: If a lower bound for the contrast ratio of the reference polarizer is known, an approximation for the contrast ratio of the polarizer under test is the arithmetic mean of calculated DUT CRs for the lower bound of the reference CR and for an ideal reference polarizer.

3. Ultra Broadband Polarizer: This plate polarizer, a product of Meadowlark Optics, Inc., combines a proprietary combination of two types of plate polarizers to achieve extinction ratios rivaling that of the Glan Thompson polarizer over most of the visible wavelength range and exceeding 1000:1 from 340 nm to 2700 nm. Apertures as large as 100 mm are possible. It has a much wider angular field of performance than the Glan Thompson and other calcite polarizers.

In Figures 5 and 6 examples of calculated contrast ratios as function of wavelength based on measurements by the authors are given for dichroic glass polarizers and ultra broadband polarizers.

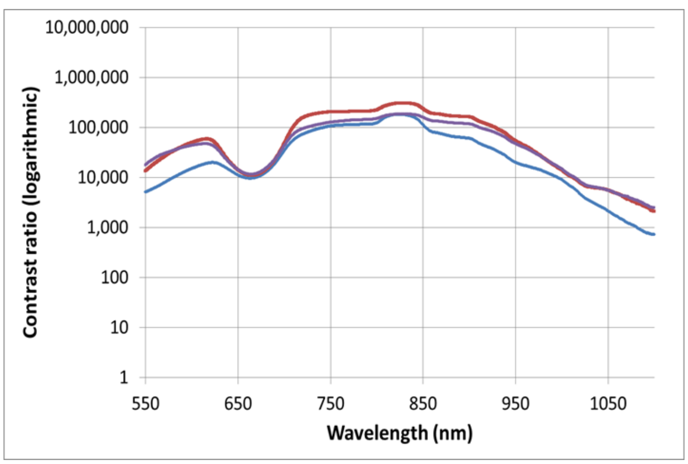

Figure 5: Calculated contrast ratio vs. wavelength of three similar dichroic glass polarizers, after measuring pair contrast ratios for the three possible combinations over the given wavelength range.

APPENDIX B. ISSUES IN MEASURING HIGH CONTRAST RATIOS

Independent of the measuring method there are the following principal hindrances to measuring contrast ratios:

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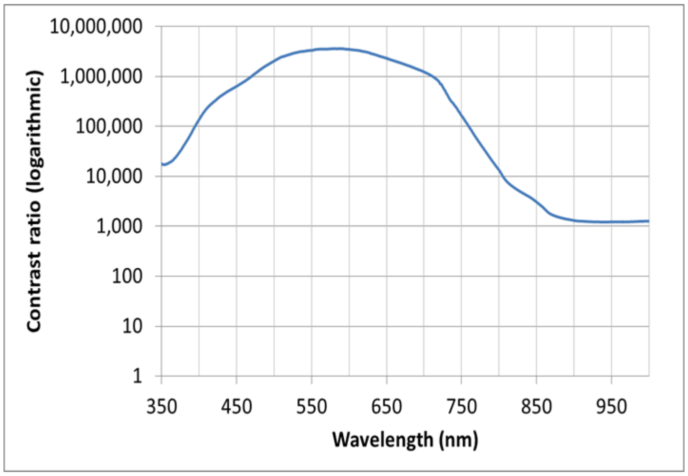

Figure 6: Contrast ratio vs. wavelength of one Ultra Broadband polarizer (manufactured by Meadowlark Optics in 2017), calculated from pair contrast ratio measurements with several similar polarizers.

1. The detector resolution is limited by signal-to-noise ratio, not allowing us to distinguish intensities that are too close to each other. Likewise the measurement of very low-intensity signals, such as the intensity for light through two crossed polarizers, requires a high integration time to lift the signal out of the environmental noise (light) and electrical noise (detector).

2. The range of measurable signals is limited by range of available integration times and maximum counts for, e.g., CCD detectors. Using a high-intensity light source, the range of measurable signals can be extended by means of neutral density filters with known attenuation vs. wavelength functions.

3. Additional (so-called instrumental) polarization might be present at the light source and through instruments such as spectrophotometers and monochromators, especially if these contain gratings. [8, 9]

4. The detected signal might depend on the polarization state of the light.

5. Detector linearity over light intensity is required to obtain the correct contrast ratio of measured intensities.

6. The beam must be collimated to obtain normal incidence on the polarizers and the detector.

7. Stray (and partially depolarized) scattered light from the light source needs to be blocked out by using an iris in front of the light source and by reducing the solid angle subtended by the detector, e.g. by moving the detector further away from the polarizers. The contrast ratio achieved by polarizers in an optical system therefore depends on the geometry of the system.

APPENDIX C. ELIMINATING INSTRUMENTAL POLARIZATION FROM TWO-POLARIZER MEASUREMENTS

As outlined in [8] we can model the combined effects of instrumental polarization by a single polarizer with amplitude attenuation coefficients $\tilde{p_x}$ and $\tilde{p_y}$, where the x axis is oriented along the major transmission axis and the y axis along the minor transmission axis of the instrumental polarization. Then, orienting two polarizers with attenuation coefficients p_{1x} and p_{1y} (where $p_{1x} > p_{1y}$) for the first and p_{2x} and p_{2y} (where $p_{2x} > p_{2y}$) for the second polarizer, the outgoing light intensity of unpolarized light entering the three-polarizer combination, is, resulting from Eq. (5),

$$I_1 = \frac{1}{2} I_0 \left[\tilde{p}_x p_{1x} p_{2x} + \tilde{p}_y p_{1y} p_{2y} \right] \tag{33}$$

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Following [10], we can also measure the outgoing transmission intensities for the other three alignment combinations, obtaining,

$$I_2 = \frac{1}{2} I_0 \left[\tilde{p}_x p_{1y} p_{2y} + \tilde{p}_y p_{1x} p_{2x}\right] \tag{34}$$

$$I_3 = \frac{1}{2} I_0 \left[\tilde{p}_x p_{1x} p_{2y} + \tilde{p}_y p_{1y} p_{2x}\right] \tag{35}$$

$$I_4 = \frac{1}{2} I_0 \left[\tilde{p}_x p_{1y} p_{2x} + \tilde{p}_y p_{1x} p_{2y}\right] \tag{36}$$

In [10] these four equations are used to calculate the unknown contrast ratios

$$R = \frac{\tilde{p}_x^2}{\tilde{p}_y^2}, \quad r_1 = \frac{p_{1x}^2}{p_{1y}^2}, \quad r_2 = \frac{p_{2x}^2}{p_{2y}^2}.$$

We prefer, avoiding subtraction of two close numbers from one another, to obtain the following two equations from Eqs. (33-36),

$$A \equiv \frac{I_1}{I_3} = \frac{R r_1 r_2 + 1}{R r_1 + r_2}, \tag{37}$$

$$B \equiv \frac{I_2}{I_4} = \frac{R + r_1 r_2}{R r_2 + r_1}, \tag{38}$$

and then to eliminate the contrast ratio R of the instrumental polarization from Eqs. (37-38):

$$r_2 = \sqrt{\frac{(A r_2 - 1)(B r_1 - 1)}{(r_1 - A)(r_1 - B)}}, \tag{39}$$

which, for $A = B = r_{12}$ (i.e. in the absence of instrumental polarization), goes over into Eq. (21) above.

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