Meadowlark Optics LCPM-3000 Liquid Crystal Polarimeter
Application Note: Determination of Retardance by Polarimetry

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1. Introduction:

The immediate purpose of a polarimeter such as the LCPM-3000 liquid crystal polarimeter from Meadowlark Optics is to measure the vector components that combine to describe the polarization state of light. A polarimeter can also be used as a precision diagnostic tool; not only is it useful for characterizing light signals and sources, it is also effective at precisely characterizing optical components through their effect on the polarization state of light. This application demonstrates a technique by which a polarimeter is used to measure the retardance of a waveplate. A retardance measurement is desirable under circumstances in which retardance must be known more precisely than the manufacturer's specification, or in which the retarder is to be used at a wavelength other than that specified by the manufacturer. For example, a waveplate specified as having a quarter-wave of retardance at 532 nm might actually have a retardance that varies by several percent at the specified wavelength, and moreover it will exhibit a significantly different (and generally unknown) retardance for 515-nm or 488-nm light.

A retarder, or waveplate, is an optical component consisting of a birefringent material that varies the polarization state of light passing through it. A retarder is characterized by a "fast" axis orthogonal to the direction of light propagation, and a "slow" or "optic" axis in the same plane and orthogonal to the fast axis. Commercial retarders are usually marked to indicate the orientation of the fast axis. The orientation of the fast axis is denoted by ρ.

The following procedure demonstrates the application of a Meadowlark Optics liquid crystal polarimeter to position a linear retarder such that its fast axis ρ is at 90°, and then to precisely measure the retardance δ of the retarder. The LCPM-3000 is a Stokes polarimeter, therefore the Mueller matrix convention for optic elements is used in the following discussion.

2. Retardance Modeling

The polarization state of light is modeled by a Stokes vector **S** consisting of four values:

$$\mathbf{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \equiv \begin{pmatrix} \text{total light intensity} \\ \text{intensity difference between horizontal \& vertical} \\ \text{intensity difference between } +45° \text{ \& } -45° \\ \text{intensity difference between right \& left circular} \end{pmatrix} \tag{1}$$

An optic that changes the polarization state of light is modeled by a Mueller matrix (4x4) such that the inner product of the Stokes vector that models the polarization state of incident light $\hat{\mathbf{S}}$ with the Mueller matrix that models the optic $\mathbf{M}$ results in a Stokes vector that represents the polarization state of light exiting the optic $\mathbf{S}$.

$$\mathbf{S} = \mathbf{M} \cdot \hat{\mathbf{S}} \tag{2}$$

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} \end{pmatrix} \cdot \begin{pmatrix} \hat{S}_0 \\ \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{pmatrix} \tag{3}$$

The Mueller matrix for a linear retarder with an undetermined retardance δ and arbitrary rotational orientation (indicated by the fast axis angle ρ is given by Kliger *et al*. as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\rho)\sin^2(\delta/2) + \cos^2(\delta/2) & \sin(4\rho)\sin^2(\delta/2) & -\sin(2\rho)\sin(\delta) \\ 0 & \sin(4\rho)\sin^2(\delta/2) & -\cos(4\rho)\sin^2(\delta/2) + \cos^2(\delta/2) & \cos(2\rho)\sin(\delta) \\ 0 & \sin(2\rho)\sin^2(\delta) & \cos(2\rho)\sin(\delta) & \cos(\delta) \end{pmatrix} \tag{4}$$

Careful selection of an incident polarization state and observation of the detected polarization vector components as the retarder is rotated makes it possible to determine the fast axis angle ρ. For this application, incident light that is linearly polarized at +45° [$\hat{\mathbf{S}} = (1, 0, 1, 0)^T$] is applied. The retarder is rotated from ρ = –90° to ρ = +90°. The S_3 component of the detected Stokes vector varies from a maximum value at $\rho = \pm 90°$ to a minimum value at $\rho=0°$ as shown in Fig. 1. The retarder can therefore be rotated to $\rho = 90°$, indicated by maximizing S_3.

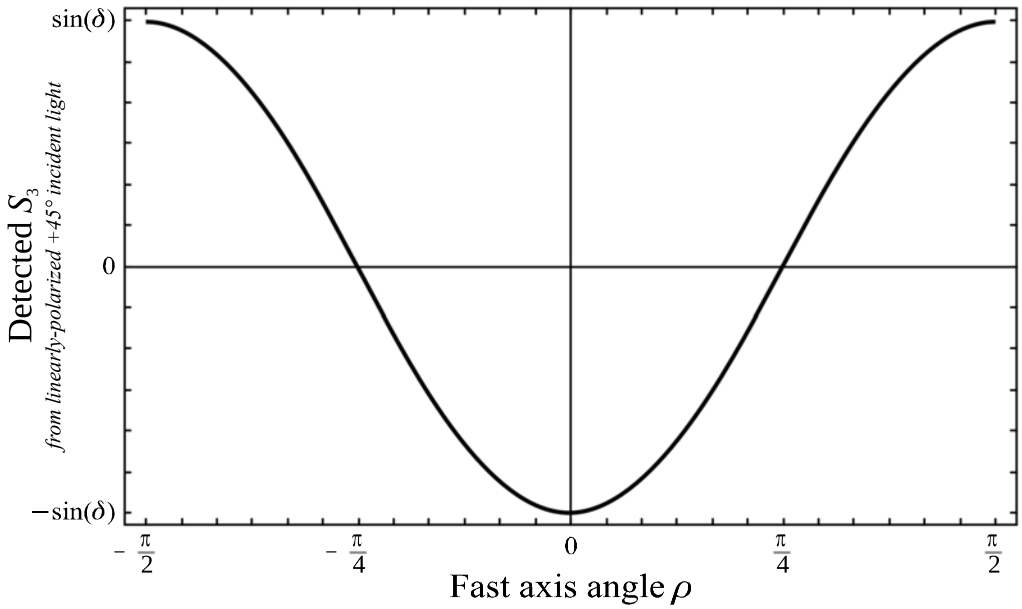

Figure 1. Detected S_3 for $-90° < \rho < 90°$ with linearly polarized incident light at +45°

Fig. 1. suggests that the retardance δ can be determined as the Arcsine of the maximized S_3; while this is one technique by which to calculate the retardance, the following procedure has less uncertainty than simply calculating Arcsine(S_3). The Mueller matrix for a linear retarder with an undetermined retardance δ, oriented at $\rho = 90°$, is given by Kliger *et al.* as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\delta) & -\sin(\delta) \\ 0 & 0 & \sin(\delta) & \cos(\delta) \end{pmatrix} \qquad (5)$$

Calculating the right side of Eq. (4) using the expression for **M** in Eq. (5) gives

$$S_0 = \hat{S}_0 \qquad (6i)$$

$$S_1 = \hat{S}_1 \qquad (6ii)$$

$$S_2 = \hat{S}_2 \cos(\delta) - \hat{S}_3 \sin(\delta) \qquad (6iii)$$

$$S_3 = \hat{S}_3 \cos(\delta) + \hat{S}_2 \sin(\delta) \qquad (6iv)$$

Eqs. (6*iii*) and (6*iv*) combine to give

$$\begin{pmatrix} S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} \hat{S}_2 & -\hat{S}_3 \\ \hat{S}_3 & \hat{S}_2 \end{pmatrix} \cdot \begin{pmatrix} \cos(\delta) \\ \sin(\delta) \end{pmatrix} \qquad (7)$$

which can be solved for tan(δ) by applying Cramer's rule

$$\tan(\delta) = \frac{\hat{S}_2 S_3 - \hat{S}_3 S_2}{\hat{S}_2 S_2 + \hat{S}_3 S_3} \qquad (8)$$

The expression in Eq. (8) can be applied to Stokes vectors measured with a Meadowlark Optics LCPM-3000 to determine the retardance δ of a retarder oriented with its fast axis at ρ = 90°.

3. Laboratory Apparatus and Procedure

The apparatus used to measure retardance consists of a light source, the retarder to be measured, and a Meadowlark Optics LCPM-3000 Stokes Polarimeter, as shown in Fig. 2. Incident light should be polarized; the specific state of polarization is arbitrary, although $\hat{S}_2$ and $\hat{S}_3$ should not both be zero, nor should $\hat{S}_2$ and $\hat{S}_3$ be configured such that S_2 and S_3 (Stokes components of light emitted from the retarder) are zero. This constraint is to prevent the denominator in Eq. (8) from approaching zero, thereby always giving a retardance of 90°. A suggested light source configuration is a laser and a polarizer positioned with its fast axis at +45° to give $\hat{S}_2$ = 1, which is the same polarization state that is used to align the fast axis of the retarder.

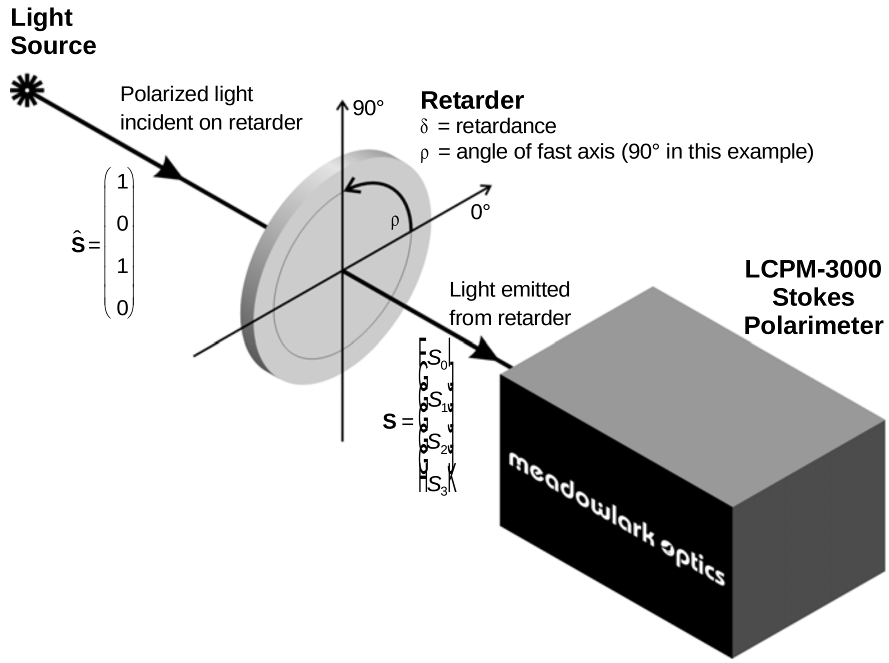

Figure 2. Optics Configuration for Retardance Measurement

The procedure for retardance measurements is as follows:

- Configure the polarization state of the light source to be linear, with a polarization angle of +45° [$\hat{\mathbf{S}}$ = (1, 0, 1, 0)^T]. This is easily accomplished by rotating a polarizer at the source and detecting the polarization state with the Meadowlark Optics LCPM-3000 liquid crystal polarimeter (Fig. 2 with the retarder removed).
- Record the precise polarization state $\hat{\mathbf{S}}$ of the incident beam detected by the Meadowlark Optics polarimeter.
- Place the retarder in the beam path.
- Rotate the retarder and watch S_3 detected by the Meadowlark Optics polarimeter. Maximizing S_3 orients the fast axis of the retarder at 90°.
- Record the precise polarization state **S** of the retarded beam detected by the Meadowlark Optics polarimeter.
- With $\hat{\mathbf{S}}$ and **S** precisely measured, use Eq. (8) to calculate the retardance δ.

4. Reference

D. S. Kliger, J. W. Lewis, and C. E. Randall. *Polarized Light in Optics and Spectroscopy*. Academic Press, Inc.: San Diego, CA. 1990.