

# Scientific and Industrial Liquid Crystal Polarimetry Applications [5741-23]

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## ABSTRACT

Since the human eye is insensitive to polarization, there is a large amount of information in many situations, which is not readily utilized. Measuring the polarization state of light is useful in many research fields including biology, chemistry, astronomy and remote sensing. The first portion of the paper discusses the simple application of accurately measuring the retardance value and fast axis position of an unknown waveplate. We will mention some of the many polarimetry applications especially in the context of non-mechanical, liquid crystal based polarimeter experimental technique. Some of these examples are from biology showing tissue birefringence changes, astronomy for solar imaging, polarimetric visualization and landmine detection.

**Keywords:** Polarimeter, polarization, birefringence, remote sensing, optical activity, retarder, waveplate, liquid crystal, Stokes, Mueller

### Variables:

- $I_H$  Intensity of the horizontal component of linearly polarized light, measured with a photodetector.
- $I_V$  Intensity of the vertical component of linearly polarized light, measured with a photodetector.
- $SOP$  State of Polarization
- $DOP$  Degree of Polarization
- $DOLP$  Degree of Linear Polarization
- $DOCP$  Degree of Circular Polarization
- $\varepsilon$  Ellipticity of a polarization ellipse
- $\alpha$  Azimuth of the major axis of a polarization ellipse, or the orientation of a linear state of polarization.
- $S$  A four-component time-invariant Stokes vector that describes the polarization state of light incident upon the polarimeter.
- $S_0$  The first Stokes component (intensity)
- $S_1$  The second Stokes component (difference between horizontal and vertical linear components)
- $S_2$  The third Stokes component (difference between  $+45^\circ$  and  $-45^\circ$  linear components)
- $S_3$  The fourth Stokes component (difference between right and left circular components)

## 1. Introduction:

There are a huge number of applications which use the polarization state of either reflected or transmitted light in order to gain information about the physical system or process with a non destructive methodology. Many books have been written on the subject so a paper this size can only scratch the surface. [1,2,3,4,5] We will review the Stokes and Mueller polarization conventions, discuss a generic retardance measurement

application, and then touch upon some particular applications. The main advantage a liquid crystal polarimeter has over a rotating waveplate device is lack of vibration and no mechanical wear issues. Only the liquid crystal molecules change position during a retardance sweep as opposed to the motors and bearing required by a rotating waveplate. Rotating devices can be equally accurate over the acromatic range of the waveplate, but a liquid crystal retarder can change retardance as function of voltage to correct for dispersion and effectively increase the useable wavelength range.

The following examples demonstrate the application of a Meadowlark Optics liquid crystal polarimeter to orient a linear retarder such that its fast axis  $\rho$  is at  $90^\circ$ , and then to precisely measure the retardance  $\delta$  of the retarder. The device used in this is a Stokes polarimeter; therefore the Mueller matrix convention for optic elements is used in the following discussion. Jones matrices (2x2) provide just as much information but are less intuitive and more difficult to discuss. [6]

## 2. Retardance Modeling:

The polarization state of light is modeled by a Stokes vector  $\mathbf{S}$  consisting of four values:

$$\mathbf{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \equiv \begin{pmatrix} \text{total light intensity} \\ \text{intensity difference between horizontal \& vertical} \\ \text{intensity difference between } +45^\circ \text{ \& } -45^\circ \\ \text{intensity difference between right \& left circular} \end{pmatrix} \quad (1)$$

which can be used to calculate the following relations:

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}, \quad DOLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}, \quad DOCP = \frac{S_3}{S_0} \quad (1a)$$

Degree of Polarization, Degree of Linear and Circular Polarization

$$\alpha = \frac{1}{2} \arctan\left(\frac{S_2}{S_1}\right), \quad \varepsilon = \frac{S_3}{1 + \sqrt{S_1^2 + S_2^2}} \quad (1b)$$

Polarization Axis Angel and ellipticity

An optic that changes the polarization state of light is modeled by a Mueller matrix (4x4) such that the inner product of the Stokes vector that models the polarization state of incident light  $\hat{\mathbf{S}}$  with the Muller matrix that models that optics  $\mathbf{M}$  results in a Stokes vector that represents the polarization state of light exiting the optic  $\mathbf{S}$ .

$$\mathbf{S} = \mathbf{M} \cdot \hat{\mathbf{S}} \quad (2)$$

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} \end{pmatrix} \cdot \begin{pmatrix} \hat{S}_0 \\ \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{pmatrix} \quad (3)$$

A retardance measurement is desirable under circumstances in which retardance must be known more precisely than the manufacturer's specification, or when the retarder is to be used at a wavelength other than that specified by the manufacturer. For example a waveplate specified as having a quarter-wave of retardance at 532 nm might actually have a retardance that varies by several percent at the specified wavelength, and moreover it will exhibit a significantly different (and generally unknown) retardance for 514-nm or 488-nm light. An alternative need for such a measurement could be required in stress analysis of transparent molded parts. Non-uniform retardance points to non-uniformities in the molding that is indicative of mechanical stress and indicates possible failure planes.

A retarder, or waveplate, is an optical component consisting of a birefringent material that varies the polarization state of light passing through it. A retarder is characterized by a "fast" axis orthogonal to the direction of light propagation, and a "slow" or "optic" axis in the same plane and orthogonal to the fast axis. Commercial retarders are usually marked to indicate the orientation of the fast axis. The orientation of the fast axis relative to horizontal polarization is denoted by  $\rho$ .

The Mueller matrix for a linear retarder with undetermined retardance  $\delta$  and arbitrary rotational orientation (indicated by the fast axis angle  $\rho$  is given by Klier *et al.* [7] as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\rho)\sin^2(\delta/2) + \cos^2(\delta/2) & \sin(4\rho)\sin^2(\delta/2) & -\sin(2\rho)\sin(\delta) \\ 0 & \sin(4\rho)\sin^2(\delta/2) & -\cos(4\rho)\sin^2(\delta/2) + \cos^2(\delta/2) & \cos(2\rho)\sin(\delta) \\ 0 & \sin(2\rho)\sin^2(\delta) & \cos(2\rho)\sin(\delta) & \cos(\delta) \end{pmatrix} \quad (4)$$

Careful selection of an incident polarization state and observation of the detected polarization vector components as the retarder is rotated makes it possible to determine the fast axis angle  $\rho$ . For this demonstration, incident light that is linearly polarized at  $+45^\circ$  [ $\hat{S} = (1, 0, 1, 0)^T$ ] is applied. The retarder is rotated from  $\rho = -90^\circ$  to  $\rho = +90^\circ$ . The  $S_3$  component of the detected Stokes vector varies from a maximum value at  $\rho = \pm 90^\circ$  to a minimum value at  $\rho = 0^\circ$  as shown in Fig. 1. The retarder can therefore be rotated to  $\rho = 90^\circ$ , indicated by maximizing  $S_3$ .

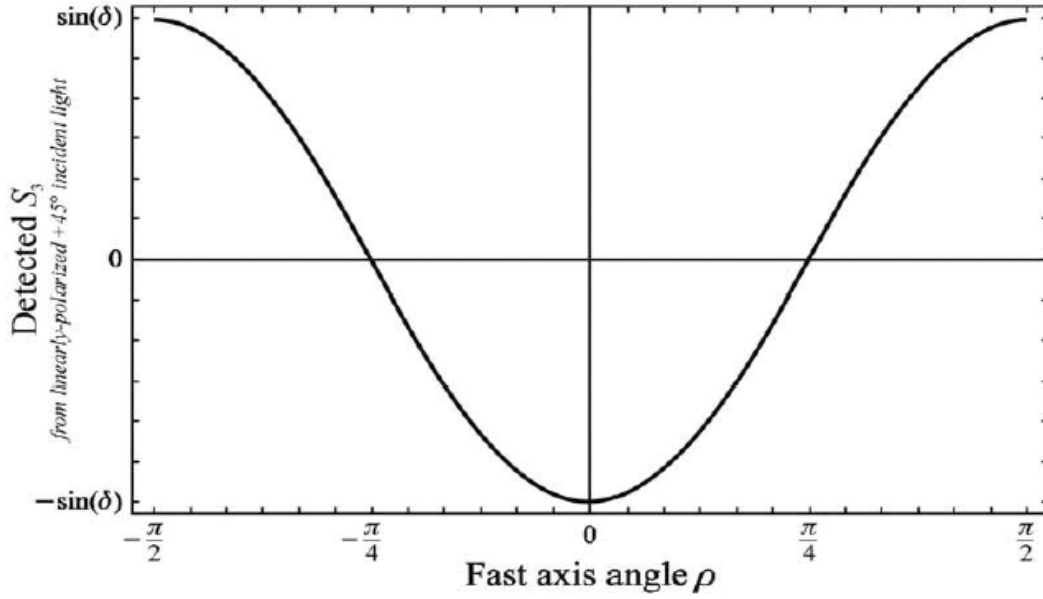


Figure1. Detected  $S_3$  for  $-90^\circ < \rho < 90^\circ$  with linearly polarized incident light at  $+45^\circ$

Fig. 1. Suggests that the retardance  $\delta$  can be determined as the Arcsine of the maximized  $S_3$ , while this is one technique by which to calculate the retardance, the following procedure has less uncertainty than simply calculating Arcsine ( $S_3$ ). The Mueller matrix for a linear retarder with an undetermined retardance  $\delta$ , oriented at  $\rho = 90^\circ$ , is given by Kliger *et al.* as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\delta) & -\sin(\delta) \\ 0 & 0 & \sin(\delta) & \cos(\delta) \end{pmatrix} \quad (5)$$

Calculating the right side of Eq. (4) using the expression for  $\mathbf{M}$  in Eq. (5) gives

$$S_0 = \hat{S}_0 \quad (6i)$$

$$S_1 = \hat{S}_1 \quad (6ii)$$

$$S_2 = \hat{S}_2 \cos(\delta) - \hat{S}_3 \sin(\delta) \quad (6iii)$$

$$S_3 = \hat{S}_3 \cos(\delta) + \hat{S}_2 \sin(\delta) \quad (6iv)$$

Eqs. (6iii) and (6iv) combine to give

$$\begin{pmatrix} S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} \hat{S}_2 & -\hat{S}_3 \\ \hat{S}_3 & \hat{S}_2 \end{pmatrix} \cdot \begin{pmatrix} \cos(\delta) \\ \sin(\delta) \end{pmatrix} \quad (7)$$

which can be solved for  $\tan(\delta)$  by applying Cramer's rule

$$\tan(\delta) = \frac{\hat{S}_2 S_3 - \hat{S}_3 S_2}{\hat{S}_2 S_2 + \hat{S}_3 S_3} \quad (8)$$

The expression in Eq. (8) can be applied to Stokes vectors measured with a polarimeter to determine the retardance  $\delta$  of a retarder oriented with its fast axis at  $p = 90^\circ$ .

### 3. Demonstration with Laboratory Apparatus and Procedure:

The apparatus used to measure retardance consists of a light source, the retarder to be measured, and a Meadowlark Optics Stokes Polarimeter, as shown in Fig. 2. Incident light should be polarized; the specific state of polarization is arbitrary, although  $\hat{S}_2$  and  $\hat{S}_3$  should not both be zero, nor should  $\hat{S}_2$  and  $\hat{S}_3$  be configured such that  $\hat{S}_2$  and  $\hat{S}_3$  (Stokes components of light emitted from the retarder) are zero. This constraint is to prevent the denominator in Eq. (8) from approaching zero, thereby always giving a retardance of  $90^\circ$ . A suggested light source configuration is a laser and a polarizer positioned with its fast axis at  $+45^\circ$  to give  $\hat{S}_2 = 1$ , which is the same polarization state that is used to align the fast axis of the retarder

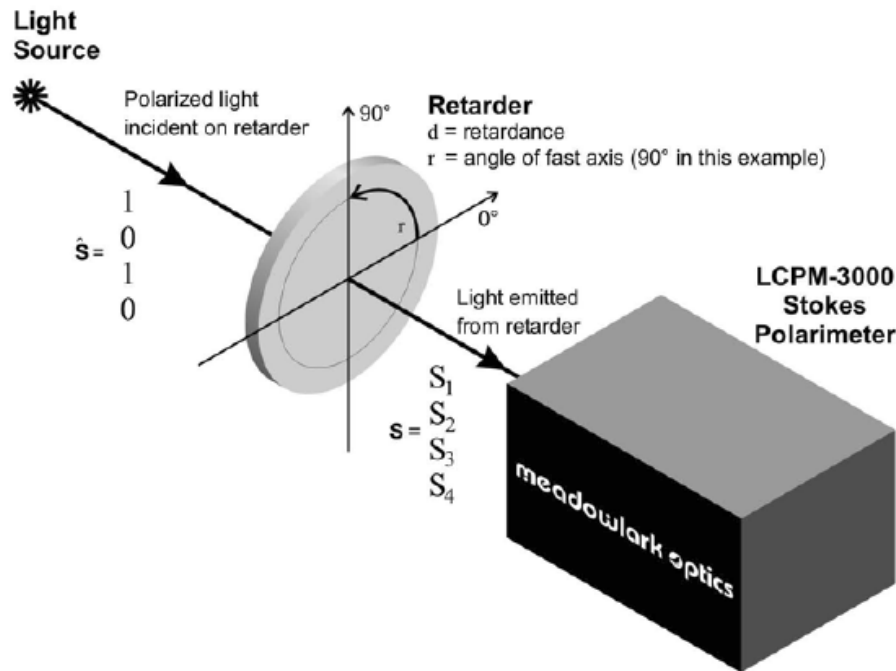


Figure 2. Optics Configuration for Retardance Measurement

The procedure for retardance measurements is as follows:

- Configure the polarization state of the light source to be linear, with a polarization angle of  $+45^\circ$  [ $\hat{S} = (1, 0, 1, 0)^T$ ]. This is easily accomplished by rotating a polarizer at the source and detecting the polarization state with the liquid crystal polarimeter (Fig. 2 with the retarder removed).

- Record the precise polarization state  $\hat{S}$  of the incident beam detected by the Meadowlark Optics polarimeter.
- Place the retarder in the beam path.
- Rotate the retarder and watch  $S_3$  detected by the polarimeter. Maximizing  $S_3$  orients the fast axis of the retarder at  $90^\circ$ .
- Record the precise polarization state  $S$  of the retarded beam detected by the Meadowlark Optics polarimeter.
- With  $\hat{S}$  and  $S$  precisely measured, use Eq. (8) to calculate the retardance  $\delta$ .

#### 4. Muller Matrix calculations:

In a similar way, the entire Muller matrix can be measured using a method of generating known (and repeatable) eigenstates while measuring the output polarization state. One approach is to mechanically generate the states either by rotating a waveplate/polarizer combination but the preferred non-mechanical system is achieved by adding two more liquid crystal cells, which are electrically driven to produce the necessary states. One aspect of this application is to achieve accurate results the incident states must completely span the polarization space. An equivalent description is that the incident states must fill the Poincaré sphere [8] in order to probe all possible output states. A counter example would be if only linear states were used, the last column in the Mueller matrix would be unsampled and unknowable.

#### 5. Scientific and Industrial Applications:

There are thousands of interesting polarimetry applications which have been investigated in the last two hundred years, but only a small handful of the more intriguing are mentioned here:

*[Meadowlark Optics does not necessarily claim any collaborative effort with the following groups, nor do we assert that these results were achieved with materials purchased from Meadowlark. They are intended to be interesting and representative examples from the huge field of polarimetry applications that exist.]*

- **Biology/Medicine – Burn depth assessment**

Using a super-luminescent LED and Polarization Sensitive Optical Coherence Tomography (PS-OCT) de Boer, Srinivas, Malekafzali, Chen and Nelson were able to image and measure the reduction in birefringence of biological tissues due to thermal damage. [9] Under normal circumstances, fibrous structures such as collagen have a measurable amount of birefringence. After thermal damage (above  $\sim 55^\circ\text{C}$ ), denatured collagen is less birefringent which may lead towards a rapid method for burn depth assessment.

- **Biology/Medicine – Imaging polarimetry in the human eye**

Dreher, Reiter and Weinred showed in 1992 that retinal thickness corresponds with the retardation value measured with a scanning Fourier ellipsometer using cadaver eyes in order to accurately measure the retinal thickness. [10] More work in 1998 by Bueno and Artal resulted in a system that can scan living retinal tissue and determine spatial variation of polarization properties. [11] Retinal damage due to retinal tissue material properties.

- **Other biological results**

A group from Chernivtsi National University in the Ukraine including Oleg Angelsky, Alexander Ushenko, Igor Mokhun and others has done some very interesting work on measuring the growth of tumor pathology of soft tissues, degenerate and dystrophic changes of connective tissue and collagenic diseases. [12] The upshot is that many human tissues (including collagen, elastin, myosin) have birefringence when healthy but show polarization changes when they suffer damage or disease. There is hope to be able to identify disease pathology in the early stages of disease and identify specific locations which may lead to better understanding of disease progression.

- **Astronomy and tunable filters**

Lyot and Solc filters have been used for a very long time but wider applications are being held back for several reasons, one of which is the accurate rotational alignment of dozens (or more) waveplates, polarizers and in some instances liquid crystal cells. By using an active alignment procedure with a real time polarimeter, manufacturing accuracy and versatility can be greatly enhanced.

As an example of Stokes polarimetry in solar astronomy a recent paper by Philip Judge, David Elmore, Bruce Lites, Christopher Keller and Thomas Rimmele discusses some of the trade offs between rotating waveplates and liquid crystal variable retarders. [13]

Also Gegg Kopp, et. Al. (now at University of Colorado at Boulder) built a tunable liquid-crystal filter for solar imaging at the helium 1083-nm line. [14] By stacking a series of tunable filters with polarizers, fixed retarders and blocking filters, they were able to achieve 0.135 nm resolutions over a free spectral width of 2.35 nm.

- **Material science**

A recent paper by Ihor Berezhnyy and Aristide Dogariu identifies different scattering mechanisms by using picosecond time resolution Mueller matrix imaging. [15] The different photon paths are obtained by imaging a series of time-resolved polarization patterns at different time delays. The overall Mueller matrix sensitivities for different scattering mechanisms are also measured which may be useful for other highly scattering systems found in biomedical and remote sensing applications. [16]

- **Visualizing hidden polarimetric information**

A group from the University of Pennsylvania including Tyo, Rowe, Pugh and Engheta has showed some wonderfully creative demonstrations of polarimetric information encoded in a variety of visual schemes. [17] By encoding Stokes and Mueller information in color and pseudo-random spot motion, visual patterns correspond to otherwise hidden polarimetry information. This method shows great promise in opening up the optical polarization dimensions into approachable and exciting applications.

- **Remote sensing – Material properties**

Gerligand, Smith and Chipman demonstrated that reflected polarized light from a brass conical target reveal considerable details about the physical geometry from a single perspective. [18] One can generalize that result into a microscopic regime where microstructures in a surface could produce useful, quantifiable information about spatial orientation.

- **Remote sensing – Landmine detection**

Barnes, M. Jones, and Bishop discuss the development of a infrared polarimetric camera system which will hopefully be able to help detect buried and surface laid mines from an aerial platform. [19]

Also Iannarilli, Scott, and S. Jones with Aerodyne have recently shown that by using polarimetric information a hyperspectral IR camera can more effectively detect landmines. [20]

## 6. Conclusions

Polarized light surrounds us in daily life but the human eye is basically insensitive to that source of information. Liquid crystal polarimeters have advantages over spinning waveplates, especially in the lack of mechanical vibration, wavelength tunability and the ability to produce arbitrary retardance combinations by simple voltage control. The most important aspect of polarimetry is to acquire this hidden information and then use it for new and innovative applications. We have mentioned demonstrations from a diverse range of scientific and industrial applications that hint at the huge, untapped information store right in front of our eyes.

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